Waves and Fields (PHYS 195): Week 5

Spring 2021 v1.5

Intro:

This week we derived the equation of motion for waves on a string – the **wave equation**. *This week we look into standing waves, harmonics, energy in waves, Fourier series, and, perhaps, start on the wave properties of sound...*

Reading:

- Friday: HRW 16.4
- Monday: HRW 16.7 and 16.2 - 16.3
- Wednesday: HRW 16.5 - 16.6

Physics Topics:

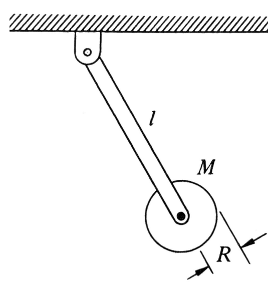
- moment of inertia
- Transverse waves
- Standing waves
- Harmonics
- Sound

Math Topics:

- Wave equation (in 1 dimension)
- Boundary conditions and phase shifts
- Superposition
- Phasors

Problems:

(1) A pendulum is built from a length ℓ of uniform rod with mass m and a disk of mass M and radius R as shown.



(thanks to K&K for the picture)

(a) What is the moment of inertia of this pendulum?

(b) What is the natural angular frequency of the pendulum? Assume that the angular amplitude of oscillation is small.

(c) Optional Bonus! How does the frequency change if the disk is on a frictionless bearing so that the disk is free to spin?

(2) Let $z = x^3y + e^{xy}$.
  (a) Compute $\frac{\partial z}{\partial x}$.
  (b) Compute $\frac{\partial z}{\partial y}$.
  (c) Compute $\frac{\partial^2 z}{\partial x \partial y}$ and $\frac{\partial^2 z}{\partial y \partial x}$. Comment on this.
  (d) Compute $\frac{\partial^2 z}{\partial x^2}$.

(3) Open the wave on a string Phet simulation.
  (a) Give the end of the string a good wiggle. Describe what happens on the right end of the string for each of the three possible boundary conditions: Fixed End, Loose End, and No End. Feel free to adjust the damping so you can more clearly see the motion.
  (b) Fix the right end, decrease the damping to zero, and add the driving force with the "Oscillate" button. Choose an amplitude of 0.18 cm and frequency of 1.67 Hz. Wait a little while. What is happening?

(4) Suppose you have a string with linear mass density 0.100 kg/m and a wave,

$$y(x,t) = 0.050\ \sin(6.00x + 12.0t),$$

propagating on this string. y and x are measured in meters and t is measured in seconds.
  (a) Is this a right moving or left moving wave?
  (b) Find (or record) k, ω, λ, f, and v.
  (c) What are the minimum and maximum speeds of the string?

(5) A sinusoidal wave traveling on a string in the negative x direction has amplitude 1.00 cm, wavelength 3.00 cm, and frequency 200 Hz. At $t = 0$, the particle of string at $x = 0$ is displaced a distance 0.80 cm above the origin and is moving upward.
  (a) Sketch the shape of the wave at $t = 0$.
  (b) Determine the function $y(x,t)$ describing the wave.

(6) HRW 16.8

(7) HRW 16.22

(8) On the derivation of the wave equation on a string:
  (a) What does $y(x,t)$ represent?
  (b) What gives the elastic restoring force for the wee bit of string of length Δx?
  (c) For these waves, what is waving?
  (d) What is the polarization?
  (e) What is the phase velocity of these waves?

Lab:

Waves on a String: Investigating harmonics, speed of propagation, and other properties of 1D waves. It will be useful to have read 16.12-16.13 before lab.

A look ahead...

Sound! HRW Chapter 17