Waves and Fields (PHYS 195): Week 7

Spring 2021 v1.5

Intro:

We'll be studying the wave properties of sound. Most of the wave behavior is the same as on waves on a string but there are some differences. One is that sound waves normally travel in three dimensions rather than one. This means that, for instance, interference can vary as you move around a room. Another is that there is a special set of units for "loudness" or intensity. Also since we can move with respect to the medium carrying sound (i.e. air), there is a new phenomena called Doppler shift.

Reading:

- Friday: HRW 17.4 - 5
- Monday: HRW 17.6 - 7

Physics Topics:

- Intensity
- Beats
- Group velocity
- Doppler Shift
- Interference in space and time
- Doppler effect non-relativistic and relativistic

Problems: Light Week Choice! Choose 6, 7, or 8 problems to complete for full credit.

(1) A "physicists' proof" of Euler's formula: There's this curious relation

$$e^{i\theta} = \cos(\theta) + i\sin(\theta)$$

where $i^2 = -1$. Let's "prove" this using Taylor series.
  (a) Find the first 4 terms of the Taylor expansion of $e^{i\theta}$ around $\theta = 0$.
  (b) Find the first 2 terms of the Taylor expansion of $\cos\theta$ around $\theta = 0$.
  (c) Find the first 2 terms of the Taylor expansion of $\sin\theta$ around $\theta = 0$.
  (d) On the basis of these expansion justify Euler's formula above.

(2) HRW 16.38

(3) Two sinusoidal waves with the same frequency travel in the same direction along a string. If

$$y(x,t) = y_1(x,t) + y_2(x,t)$$

with

$$y_1(x,t) = y_{m_1}\sin(kx - \omega t + \varphi_1) \text{ and } y_2(x,t) = y_{m_2}\sin(kx - \omega t + \varphi_2)$$

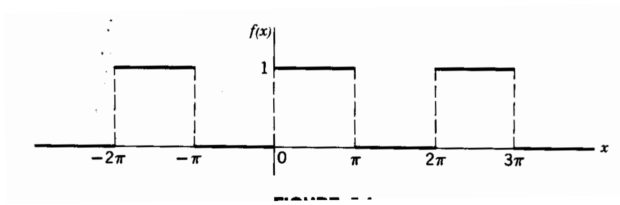
Let $y_{m1} = 3.5$ cm, $y_{m2} = 4.2$ cm, $\varphi_1 = 0$ and $\varphi_2 = \pi/4$,
  (a) what is the resulting wave's amplitude?
  (b) and phase?

(4) Go to the Phet Fourier series simulator. Play with the Wave Game for as bit to see how is works.
  (a) Choose level 7. Sketch or screenshot the pink curve.

(b) What is the Fourier series for this curve? (At least for a little while resist the temptation of clicking "Cheat", which occurs after the hints.) Write out the sum of the waves in terms of the terms $\sin(n\pi x/L)$. This is your Fourier sine series for the pink curve.

For these demos $L = 0.39$ m.

(5) *Fourier series by calculation* For the square wave on $(-\pi, \pi)$:



(Let $L = \pi$.)

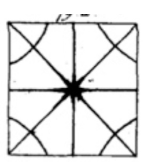
(a) Find the Fourier coefficients by integrating the equations for the coefficients. To help you get started the function looks "sin-ish" so let's start with the b_n's

$$\begin{aligned} b_n &= \frac{1}{L}\int_{-L}^{L} f(x)\sin\left(\frac{n\pi x}{L}\right) dx \\ &= \frac{1}{\pi}\int_{-\pi}^{0} (0)\sin(nx)\, dx + \frac{1}{\pi}\int_{0}^{\pi} (1)\sin(nx)\, dx \\ &= \frac{1}{\pi}\int_{0}^{\pi} \sin(nx)\, dx \end{aligned}$$

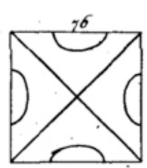
Now integrate to obtain the result. Setting up the calculation of the a_n's is similar.

(b) Using these coefficients, write the first four non-vanishing terms in the Fourier series.

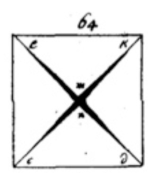
(6) Order the following sketches by Chladni by frequency with the first in the list having the lowest resonant frequency and the last having the highest frequency. Explain your reasoning. Your explanation is the most important part of the solution.



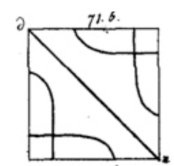
(a.)



(b.)



(c.)



(d.)

(7) Variable stars such as "delta Cephei" in the constellation Cepheus have periodic changes in their brightness. (You might observe these in the sky on a clear night.) For some of these stars the periods determined by the first standing wave of longitudinal oscillations in the radial direction. Informally they are sometimes called 'breathing modes' since the star expands and contracts as if it were breathing. The star's radial oscillations are correlated with variations in brightness. The surface of the star is a displacement anti-node. Let's call the radius R and the speed of 'sound' in the star v.

(a) Explain why you might expect that the center of the star is a displacement node.

(b) Make a sketch of the star and the harmonic or 'mode'. Feel free to use a transverse wave picture.

(c) What is the algebraic expression for the period of the mode?

(d) An approximation of the physics gives

$$v \simeq \sqrt{\frac{GM}{R}}$$

Find the speed of sound for a star with $M = 1.4 \times 10^{32}$ kg and average radius of $R = 4.5 \times 10^{10}$ m.

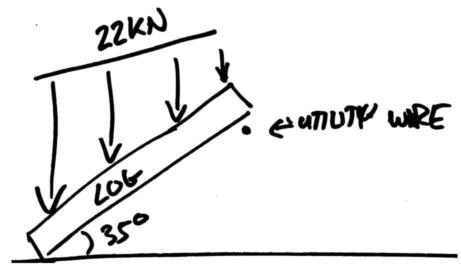
(e) Find the period of the fundamental mode in (earth) days for this star.

(8) HRW 17.107

(9) **Bonus** Arborist Erik Sveum in Vancouver (WA) contacted me about the following question. "I'm an arborist for a large company that contracts with utility providers to remove vegetation near conductors, e.g. prune and remove trees from power lines. Occasionally, we are called upon to remove trees or parts of trees that have fallen across conductors [see (a.) below]. After the linemen provide proper electrical grounding, we then cut small pieces off the tree until it eventually falls from the lines. We try as best we can to secure the wires with ropes to prevent the sections of wood from being thrown in a dramatic fashion. We usually remove as much of the top/overhanging portion of the tree as we can, then remove pieces from the bottom/butt end of the log until we can pull the tree off the wires. ... **I [want] to determine the force a given weight (log) applies to the wire and the ground at a given angle and the relationship as the weight and angle changes.** I need to know this in order to determine the best angle at which to apply tension with the rope as well as how much tension to apply to the rope. We can calculate for the weight of the log via green weight log chart and we can get the angle of the log from a simple measurement. ... The ultimate goal is to remove/drag the log off the line with minimal rebound to the lines/damage to the utility pole and of course not exposing ourselves to all of that potential/kinetic energy as it is released."

Can you help Erik out? If so, start by finding the normal force applied by the tree (assuming it is a cylinder with a uniform density and radius) and then work up a method to attach cable(s) and solve the problem.

(a.)



(b.)

Lab:

Speed of sound

A look ahead...

Doppler shift and other fun aspects of sound - See later sections of Chapter 17