"We all know what light is,
but it is not as easy to tell what light is"

- Samuel Johnson

Light as a Wave

9 April 2007

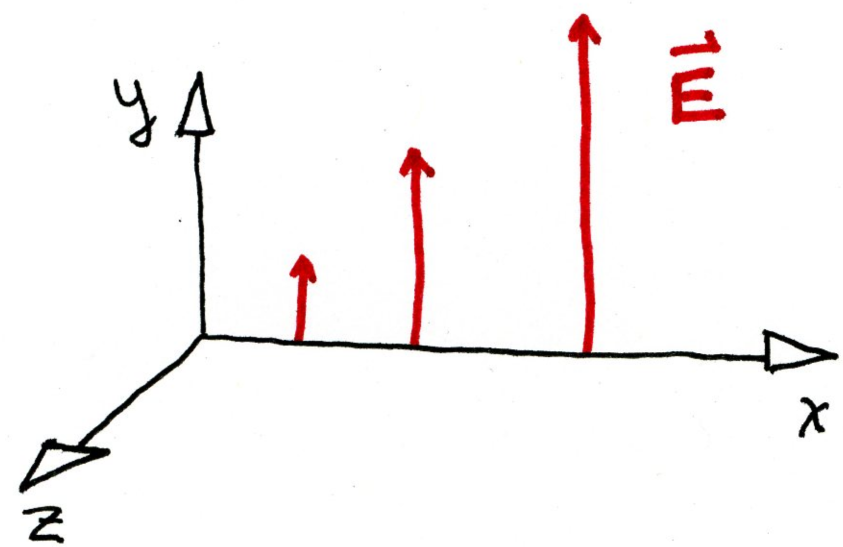
What is the relation between **E** & **B** Dynamics and Waves?

Light: A Derivation

We'll look at a wave moving along the x-axis. Suppose the electric field is along the y axis so that

$$\mathbf{E} = E_y(x,t)\hat{j}$$

Suppose it is increasing in magnitude as

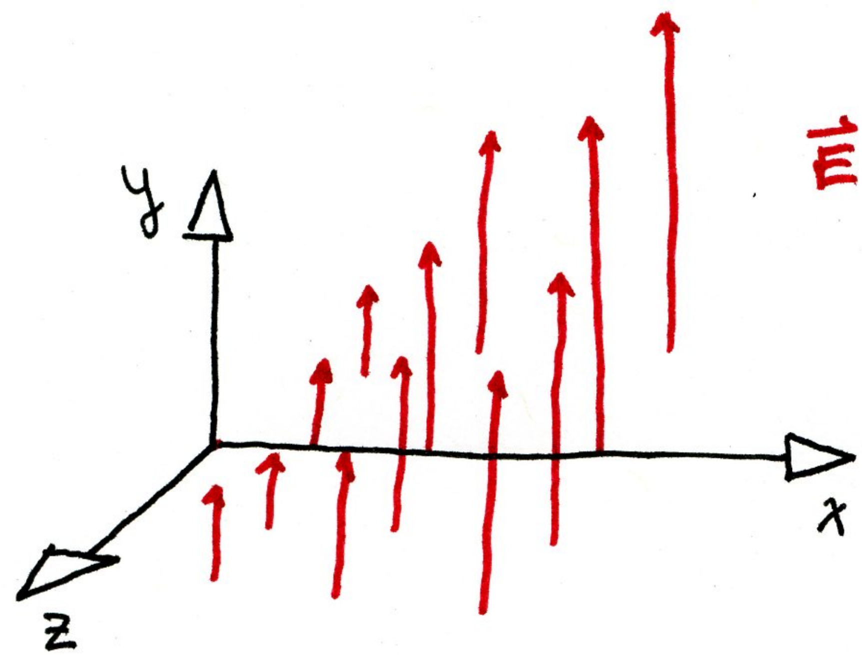


Light: A Derivation

We'll look at a wave moving along the x-axis. Suppose the electric field is along the y axis so that

$$\mathbf{E} = E_y(x,t)\hat{j}$$

Or more accurately

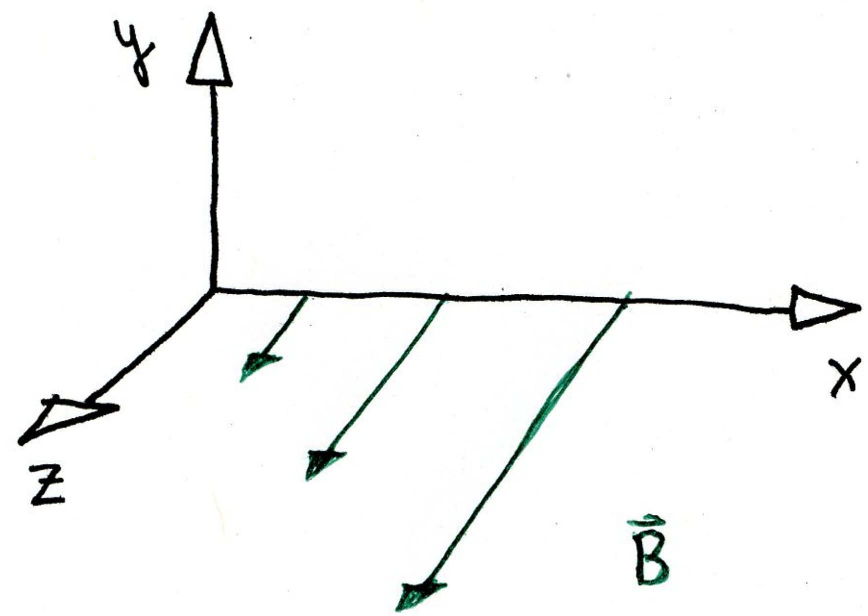


Light: A Derivation

Let's suppose that the magnetic field is along the z axis so that

$$\mathbf{B} = B_z(x,t)\widehat{k}$$

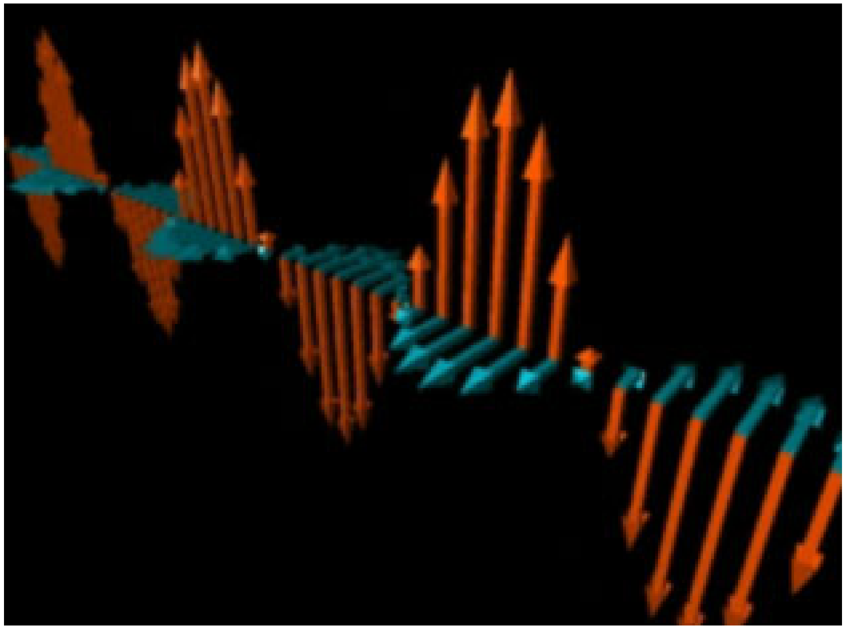
It also increases in the x direction so that



Light: A Derivation

In this case the dynamics of the electro-magnetic field gives

$$\boxed{\frac{\partial B_z}{\partial x} = -\mu_o \epsilon_o \frac{\partial E_y}{\partial t}} \quad \text{and} \quad \boxed{\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}}$$



Light: A Derivation

To combine these equations take the x-derivative of this last equation

$$\frac{\partial^2 E_y}{\partial x^2} = -\frac{\partial^2 B_z}{\partial t \partial x}$$

and use the first to find

$$\frac{\partial^2 B_z}{\partial x \partial t} = - - \mu_o \epsilon_o \frac{\partial^2 E_y}{\partial^2 t}$$

Light: A Derivation

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Hence,

$$\boxed{\frac{\partial^2 E_y}{\partial x^2} = \mu_o \epsilon_o \frac{\partial^2 E_y}{\partial t^2}}$$

A Wave Equation! (Similarily for B_z.) Thus,

$$v = \frac{1}{\sqrt{\mu_o \epsilon_o}} = 2.99 \times 10^8 m/s$$

Speed of Light:

1600 Galileo attempts to measure the speed of light by uncovering lanterns!

"If not instantaneous, it is extraordinarily rapid."

1676 Ole Römer (Cassini) measures change in eclipse times of Io, a moon of Jupiter "2 a.u. in 22 minutes" or

$$v \approx 2.3 \times 10^8 m/s$$

Really close! First measurement of a universal constant!