"We all know what light is,
but it is not as easy to tell what light is"

- Samuel Johnson

Light as a Wave

17 April 2009

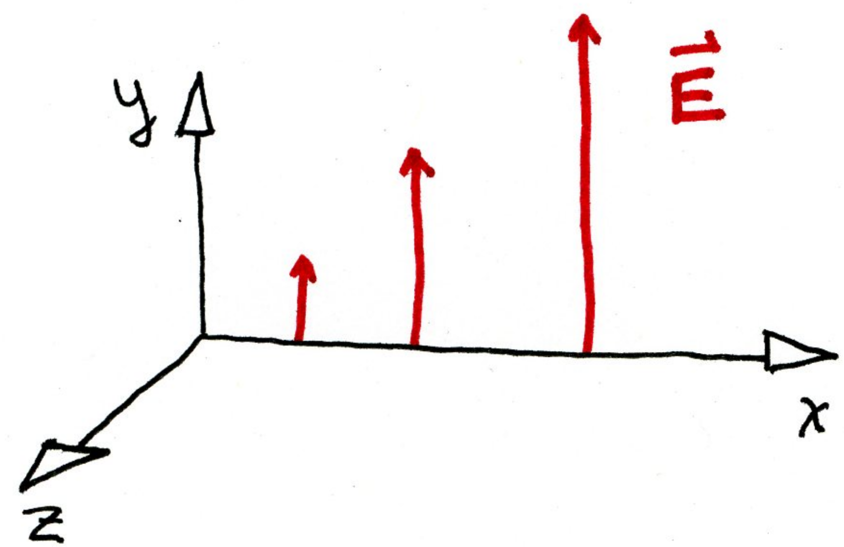
What is the relation between **E** & **B** Dynamics and Waves?

Light: A Derivation

We'll look at a wave moving along the x-axis. Suppose the electric field is along the y axis so that

$$\mathbf{E} = E_y(x,t)\hat{j}$$

Suppose it is increasing in magnitude as

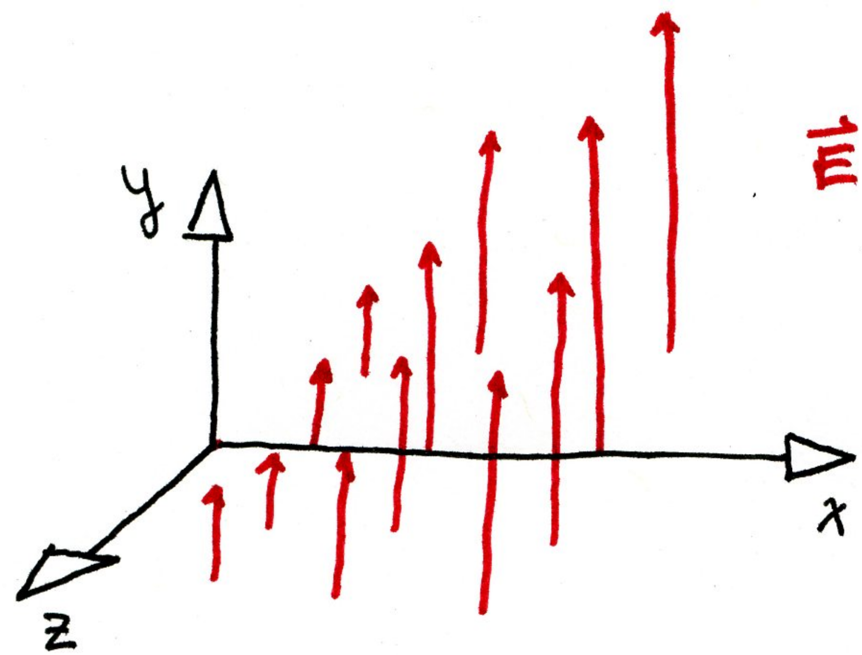


Light: A Derivation

We'll look at a wave moving along the x-axis. Suppose the electric field is along the y axis so that

$$\mathbf{E} = E_y(x,t)\hat{j}$$

Or more accurately

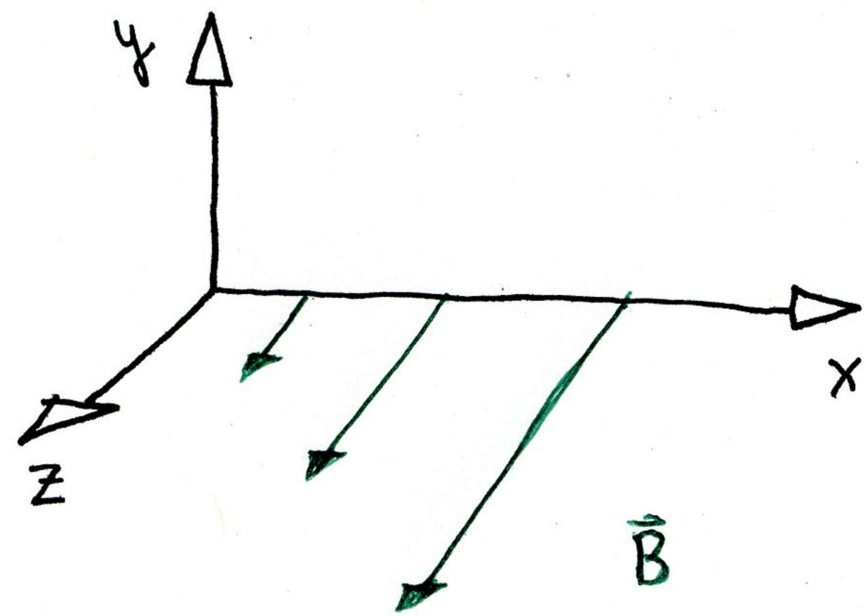


Light: A Derivation

Let's suppose that the magnetic field is along the z axis so that

$$\mathbf{B} = B_z(x,t)\widehat{k}$$

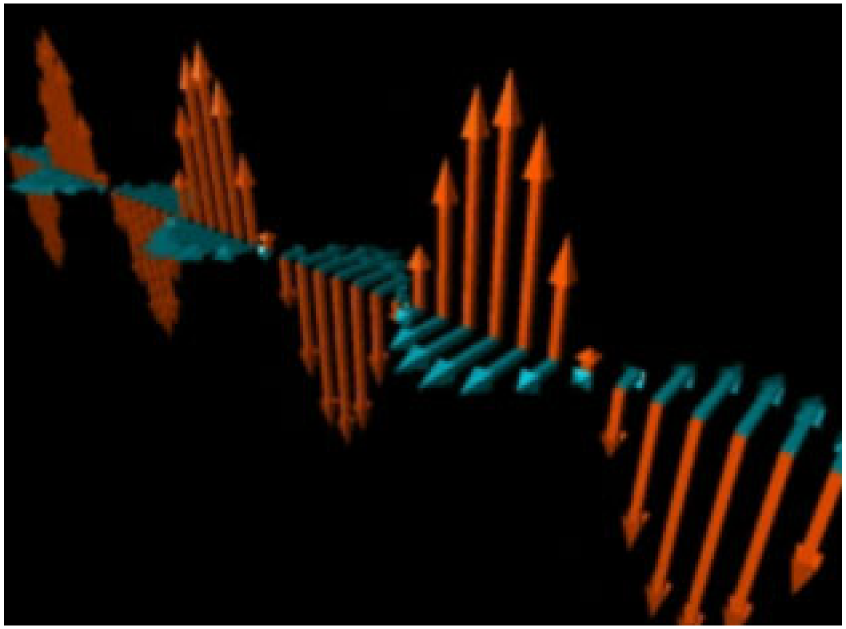
It also increases in the x direction so that



Light: A Derivation

In this case the dynamics of the electro-magnetic field gives

$$\boxed{\frac{\partial B_z}{\partial x} = -\mu_o \epsilon_o \frac{\partial E_y}{\partial t}} \quad \text{and} \quad \boxed{\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}}$$

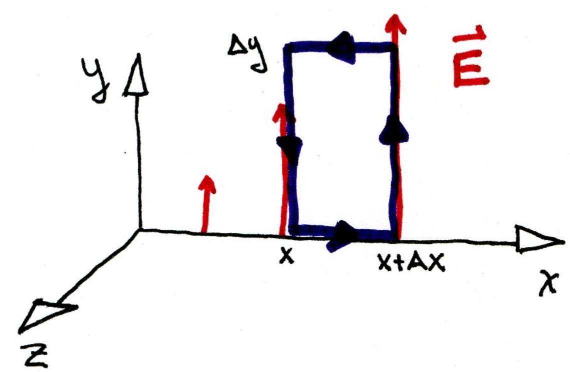


Light: A Derivation

In more detail: Now choose a wee Faradayan loop of length Δy and width Δx. Faraday's law is

$$\oint \mathbf{E} \cdot d\ell = -\frac{d}{dt} \int \mathbf{B} \cdot d\mathbf{A}$$

$$\oint \mathbf{E} \cdot d\ell = E_y(x + \Delta x)\Delta y - E_y(x)\Delta y$$



Light: A Derivation

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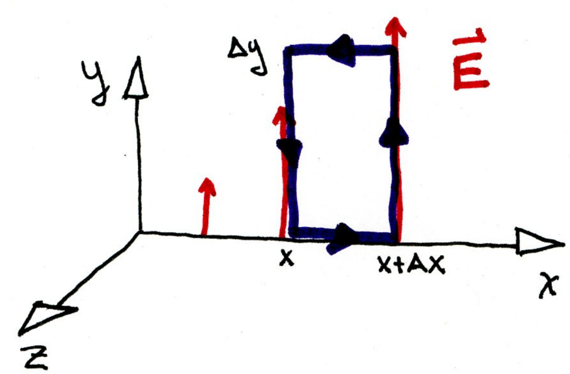
Expanding with Taylor gives

$$E_y(x + \Delta x) \approx E_y(x) + \frac{\partial E_y}{\partial x}\Delta x$$



Light: A Derivation

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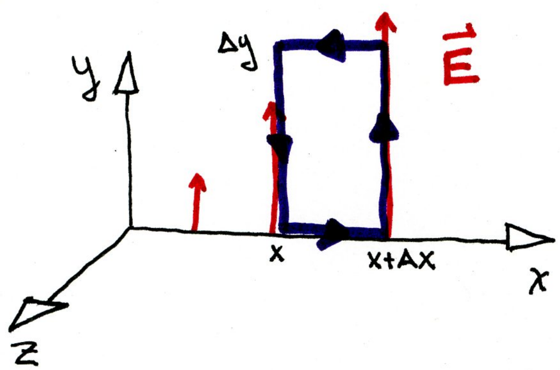
$$\oint \mathbf{E} \cdot d\ell = E_y(x + \Delta x)\Delta y - E_y(x)\Delta y$$

Hence,

$$\oint \mathbf{E} \cdot d\ell \approx \frac{\partial E_y}{\partial x} \Delta x \Delta y$$



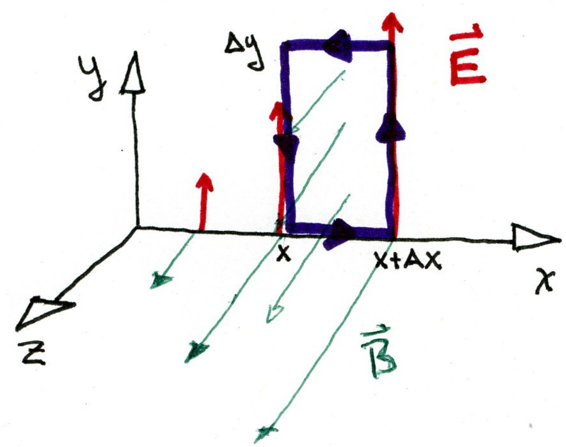
Light: A Derivation

Now choose a wee Faradayan loop of length Δy and width Δx. Faraday's law is

$$\oint \mathbf{E} \cdot d\ell = -\frac{d}{dt} \int \mathbf{B} \cdot d\mathbf{A}$$

Meanwhile, there is also magnetic flux so

$$-\frac{\partial}{\partial t} \int \mathbf{B} \cdot d\mathbf{A} = -\frac{\partial}{\partial t} \int B_z dA$$



Light: A Derivation

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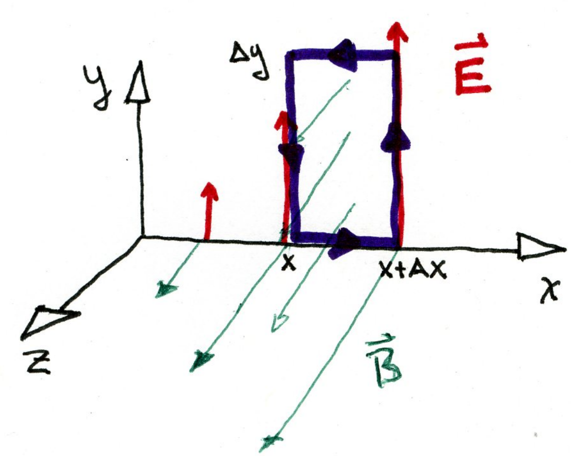
For a wee loop,

$$-\frac{\partial}{\partial t} \int \mathbf{B} \cdot d\mathbf{A} \approx -\frac{\partial B_z}{\partial t} \Delta x \Delta y$$



Light: A Derivation

Now choose a wee Faradayan loop of length Δy and width Δx. Faraday's law is

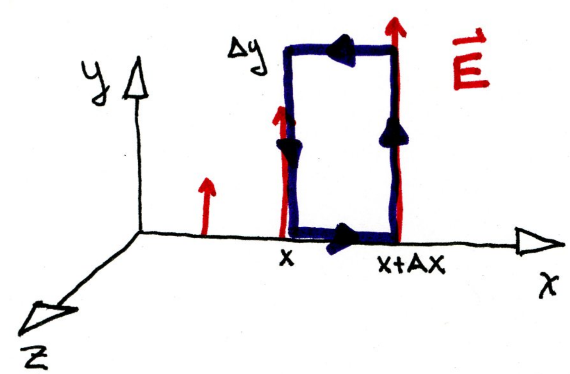
$$\oint \mathbf{E} \cdot d\ell = -\frac{d}{dt} \int \mathbf{B} \cdot d\mathbf{A}$$

Combining these last two results we find

$$\frac{\partial E_y}{\partial x} \Delta x \Delta y = -\frac{\partial B_z}{\partial t} \Delta x \Delta y$$

Or,

$$\boxed{\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}}$$



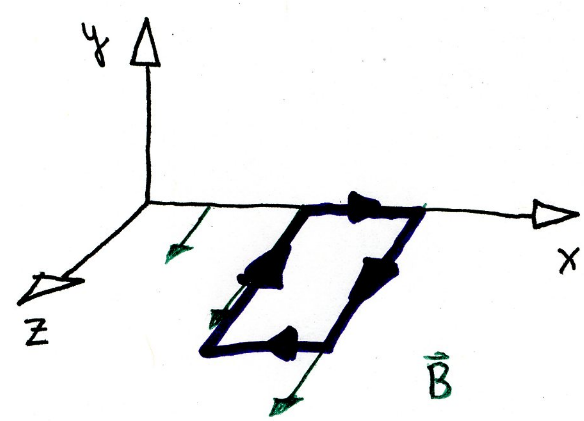
Light: A Derivation

Oh but there's electric flux as well! Choose another wee loop of length Δz and width Δx. Maxwell's equations give

$$\oint \mathbf{B} \cdot d\ell = +\mu_o \epsilon_o \frac{d}{dt} \int \mathbf{E} \cdot d\mathbf{A}$$

We find

$$\oint \mathbf{B} \cdot d\ell \approx \frac{\partial B_z}{\partial x} \Delta x \Delta z$$



And

$$\boxed{\frac{\partial B_z}{\partial x} = -\mu_o \epsilon_o \frac{\partial E_y}{\partial t}}$$

Light: A Derivation

To combine these equations take the x-derivative of this last equation

$$\frac{\partial^2 E_y}{\partial x^2} = -\frac{\partial^2 B_z}{\partial t \partial x}$$

and use the first to find

$$\frac{\partial^2 B_z}{\partial x \partial t} = - - \mu_o \epsilon_o \frac{\partial^2 E_y}{\partial^2 t}$$

Hence,

$$\boxed{\frac{\partial^2 E_y}{\partial x^2} = \mu_o \epsilon_o \frac{\partial^2 E_y}{\partial t^2}}$$

A Wave Equation! (Similarily for B_z.) Thus,

$$v = \frac{1}{\sqrt{\mu_o \epsilon_o}} = 2.99 \times 10^8 m/s$$

Speed of Light: Measurments

1600 Galileo attempts to measure the speed of light by uncovering lanterns!

"If not instantaneous, it is extraordinarily rapid."

1676 Ole Römer (Cassini) measures change in eclipse times of Io, a moon of Jupiter "2 a.u. in 22 minutes" or

$$v \approx 2.3 \times 10^8 m/s$$

Really close! First measurement of a universal constant!