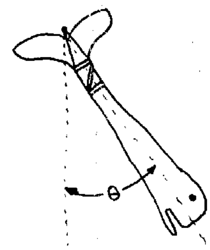
(5) **(10 pts.)** A whale undergoing back therapy gently swings as shown. Assuming that the whale has a moment of inertia I and the distance from the point of rotation to the center of mass is ℓ, derive the whale's equation of motion:



(a) In the position shown find the free body diagram for the whale
(b) Using $\tau = I\alpha$ find the equations of motion.
(c) Assuming the angle θ is less than 20^o, find the equation of motion.
(d) What type of motion is this?

(a)

(b) $\tau = -mgl\sin\theta = I\alpha = I\dfrac{d^2\theta}{dt^2}$

$$\Rightarrow \frac{d^2\theta}{dt^2} + \frac{mgl}{I}\sin\theta = 0$$

(c) Now $\sin\theta \simeq \theta$ so $\dfrac{d^2\theta}{dt^2} + \dfrac{mgl}{I}\theta = 0$

(d) SHM! with $\omega_0 = \sqrt{\dfrac{mgl}{I}}$

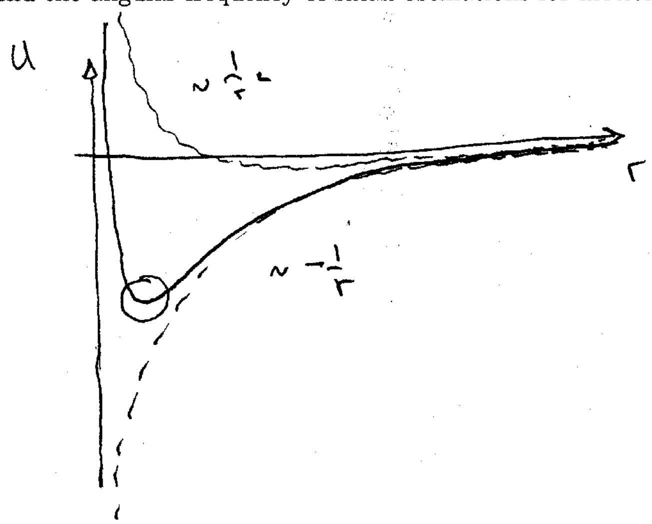
(6) (10 pts.) Consider the effective potential energy given by

$$U(r) = -\frac{\lambda}{r} + \frac{L^2}{2mr^2}$$

for a planet of mass m and angular momentum L. The constant λ characterizes the gravitational potential. For the purposes of this problem you just assume L, m, and λ are constants.

(a) Sketch the potential. One method is to first sketch each term on the same graph then graphically add the curves.
(b) Find the stable equilibrium point.
(c) Find the first 3 terms of the Taylor expansion around the equilibrium point.
(d) Find the angular frequency of small oscillations for motion around this point.

(a)



(b) $$\frac{dU}{dr} = +\frac{\lambda}{r^2} - \frac{L^2}{mr^3} = 0 \Rightarrow \frac{\lambda}{1} = \frac{L^2}{mr_0} \Rightarrow r_0 = \frac{L^2}{\lambda m}$$

(c) $$U(r) = U(r_0) + \left.\frac{dU}{dr}\right|_{r_0}(r-r_0) + \frac{1}{2}\left.\frac{d^2U}{dr^2}\right|_{r_0}(r-r_0)^2 + \cdots$$

$$= \left(-\frac{\lambda}{\frac{L^2}{\lambda m}} + \frac{\cancel{L^2}}{\frac{2m L^{\cancel{4}2}}{\lambda^2 m^2}}\right) + \left[\frac{\lambda^3 m^2}{L^4} - \frac{L^2\lambda^3 m^3}{mL^6}\right](r-r_0)$$

$$+\left(\frac{1}{2}\right)\left[-\frac{2\lambda}{r^3} + \frac{3L^2}{mr^4}\right]_{r_0}(r-r_0)^2 + \cdots$$

$$= \left(-\frac{\lambda^2 m}{L^2} + \frac{\lambda^2 m}{2L^2}\right) + \cancel{\left[\frac{\lambda^3 m^2}{L^4} - \frac{\lambda^3 m^2}{L^4}\right]}^{\,0}(r-r_0) + \left(\frac{1}{2}\right)\left[-\frac{2\lambda^4 m^3}{L^6} + \frac{3\cancel{L^2}\lambda^4}{\cancel{m}L^{\cancel{8}6}}\right] \times (r-r_0)^2$$

$$= -\frac{\lambda^2 m}{2L^2} + \frac{1}{2}\underbrace{\left(\frac{\lambda^4 m^3}{L^6}\right)}_{k_{eff}}(r-r_0)^2 \quad \Rightarrow \text{(d)} \quad \omega_0 = \sqrt{\frac{k_{eff}}{m}} = \frac{\lambda^2 m}{L^3}$$

(7) **(10 pts.)** A lyre, or simple harp, is made of a wood frame and a set of strings. Suppose that one nylon string is tuned to D_4 at 294 Hz, has a length of 0.336 m, and has a linear mass density of 7.2 g m^{-1}.

(a) Why is it that when you pluck a string that you hear the 294 Hz standing wave (and higher harmonics)?

(b) Find the wave length of the fundamental mode.

(c) What is the tension in the string? State any assumptions your make.

(d) Find the distance between the nodes for the 3rd harmonic.

(e) What is the phase velocity for this string?

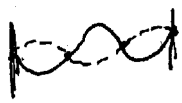
(a.) THESE HARMONICS ARE AMPLIFIED THROUGH CONSTRUCTIVE INTERFERENCE. RESONANCE!

(b.)  $\lambda_1 = 2L \simeq 0.672\ m \simeq 0.67\ m$

(c.) $V = \lambda_1 f_1 = \sqrt{\frac{F_T}{\mu}}$ SO $\lambda_1^2 f_1^2 \mu = F_T$

$\Rightarrow F_T \simeq (.672)^2 (294)^2 (7.2 g\ m^{-1}) \simeq 281\ N$! TOO HIGH?

$\simeq 280 N$

(d)  $X_{NODES} = \frac{L}{3} \simeq 11.2\ cm \simeq 11\ cm$

(e) $V \simeq \lambda_1 f_1 \simeq 198\ m/s \simeq 200\ m/s$

(f) POOR ASSUMPTION, MAYBE ~~D_4~~ D_4 IS SECOND HARMONIC SO THE TENSION IS MORE LIKE 70 N?