**Oscillations and Waves Review**

Mostly in 1 spatial dimension. For any system around equilibrium,

\[ U(x) = U(x_0) + \frac{dU}{dx} x_0 + \frac{1}{2} \frac{d^2U}{dx^2} (x-x_0)^2 \]

Taylor Exp.

\[ U(x) = \frac{1}{2} k x^2 \]

Results in

**Simple Harmonic Motion**:

\[ E = \frac{1}{2} k x^2 = \frac{1}{2} m v^2 + \frac{1}{2} k x^2 \]

EOM:

\[ \frac{d^2x}{dt^2} + \omega_0^2 x = 0 \]

Initial conditions set \((x, v)\). Solutions:

\[ x(t) = A \cos(\omega t + \phi) \]

- **Add Damping**:

\[ F_d = -b \frac{dx}{dt} \]

\( \rightarrow \) 3 cases:

- Underdamped
- Critical Damped
- Overdamped

Solutions:

\[ x(t) = A e^{-\alpha t} \cos(\omega t + \phi) \]

- **Add Driving**:

EOM:

\[ \frac{d^2x}{dt^2} + \frac{b}{m} \frac{dx}{dt} + \omega_0^2 x = \frac{F_0}{m} \cos(\omega t) \]

Steady state solution has:

\[ x(t) = A(t) \cos(\omega t - \delta) \]

Amplitude:

\[ A \approx \frac{F_0}{m \omega_0} \]

Phase:

\[ \delta = \frac{\pi}{2} \]

Many systems interact with environment and waves.

EOM:

\[ \frac{\partial^2 D}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 D}{\partial t^2} \]

V = \sqrt{\frac{D}{\mu}}

For strings, phase velocity of wave:

Superposition:

Standing waves:

\[ f = \frac{n}{2L} \]

Average energy transport:

\[ \frac{1}{2} \int_0^L \int_0^T D(t) \, dx \, dt \]