For simplicity, we'll work in a pipe, i.e. in 1D.

\[ v = Ax \]

We'll study a section of fluid \( Ax \) long. The 'ends' of this section are surfaces at \( x \) and \( x + Ax \). These surfaces are made of particles. We'll look at the equation of motion of their displacement \( D(x) \) and \( D(x + Ax) \). Let's suppose that the fluid has a density of \( \rho \).

Thus, this section has a mass

\[ M = \rho V = \rho Ax. \]

Suppose we increase the pressure on the section. Its volume \( \Delta V = A Ax \) will change as

\[ \Delta V = A \left[ D(x + Ax) - D(x) \right] \]

Doing a Taylor expansion of the first term

\[ \approx A \left[ D(x) + \frac{\partial D}{\partial x} Ax - D(x) \right] \]

\[ = A \frac{\partial D}{\partial x} Ax \]

\text{(1)}
THE PICTURE OF THE SECTION IS THIS

\[ \Delta x \]

\[ F(x) \quad \text{to} \quad F(x+\Delta x) \]

NEWTON TELLS US THAT

\[ M\Delta v = \sum F \quad \text{or} \]

\[ pA\Delta x \frac{2^2D}{2t^2} = -\left( F(x+\Delta x) - F(x) \right) \quad \text{Expanding} \quad F(x+\Delta x) \]

\[ = -\left( \frac{dF}{dx} \Delta x \right) \]

\[ = -\frac{dF}{dx} \Delta x \]

\[ \Rightarrow \quad pA \frac{2^2D}{2t^2} = -\frac{dF}{dx} \quad (2) \]

NOW WE NEED TO FIGURE OUT WHAT "F" IS!

WE KNOW THAT

\[ \Delta P = -\frac{B}{V} \Delta V \]

SO

\[ F = A\Delta P = -\frac{BA}{V} \Delta V \]
Hence from Eqn (1)

\[ F = -\frac{BA^2}{V} \frac{\partial D}{\partial x} \Delta x \]

Taking \( \frac{\partial}{\partial x} \) then \( \frac{\partial F}{\partial x} = -\frac{BA^2}{V} \frac{\partial^2 D}{\partial x^2} \Delta x \)

Using \( V = AAx \) and Eqn (2) we find

\[ PA \frac{\partial^2 D}{\partial t^2} = -\left( -\frac{BA^2}{AAx} \frac{\partial^2 D}{\partial x^2} \Delta x \right) \]

\[ \therefore \frac{\partial^2 D}{\partial x^2} = \left( \frac{P}{B} \right) \frac{\partial^2 D}{\partial t^2} \rightarrow V = \sqrt{\frac{B}{P}} \]

The wave Eqn for \( P = D(x,t) \) ! Sound is a wave that travels at \( V = \sqrt{\frac{B}{P}} \).

Note:
- Solids same as before
- Superposition "
- Energy "