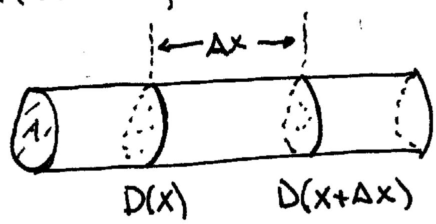
PHYS 195: WAVE EQU'N AND SOUND

FOR SIMPLICITY, WE'LL WORK IN A PIPE, I.E. IN 1D



$V = A\Delta X$

LIKE AIR OR WATER

WE'LL STUDY A SECTION OF FLUID ΔX LONG. THE 'ENDS' OF THIS SECTION ARE SURFACES AT X AND $X + \Delta X$. THESE SURFACES ARE MADE OF PARTICLES. WE'LL LOOK AT THE EQUATION OF MOTION OF THEIR DISPLACEMENT $D(X)$ AND $D(X+\Delta X)$. LET'S SUPPOSE THAT THE FLUID HAS A DENSITY OF ρ. THUS, THIS SECTION HAS A MASS

$$m = \rho V = \rho A \Delta X.$$

SUPPOSE WE INCREASE THE PRESSURE ON THE SECTION. ITS VOLUME $V = A\Delta X$ WILL CHANGE AS

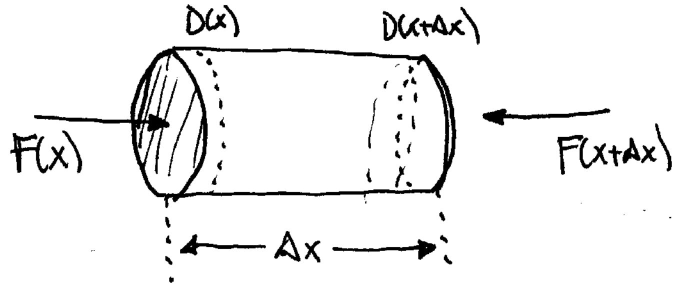
$$\Delta V = A\left[D(X+\Delta X) - D(X)\right]$$

DOING A TAYLOR EXPANSION ON THE FIRST TERM

$$\simeq A\left[D(X) + \frac{\partial D}{\partial X}\Delta X - D(X)\right]$$

$$= A\frac{\partial D}{\partial X}\Delta X \qquad (1)$$

THE PICTURE OF THE SECTION IS THIS



NEWTON TELLS US THAT

$$ma = \Sigma F \quad \text{OR}$$

$$\rho A \Delta x \frac{\partial^2 D}{\partial t^2} = -\Big(F(x+\Delta x) - F(x) \Big) \qquad \text{EXPANDING } F(x+\Delta x)$$

$$\simeq -\Big(F(x) + \frac{\partial F}{\partial x} \Delta x - F(x) \Big)$$

$$= -\frac{\partial F}{\partial x} \Delta x.$$

$$\Rightarrow \rho A \frac{\partial^2 D}{\partial t^2} = -\frac{\partial F}{\partial x} \qquad (2).$$

NOW WE NEED TO FIGURE OUT WHAT "F" IS!

WE KNOW THAT

$$\Delta P = -\frac{B}{V} \Delta V$$

SO

$$F = A \Delta P = \frac{-BA}{V} \Delta V$$

HENCE FROM EQU'N (1)

$$F = -\frac{BA^2}{V}\frac{\partial D}{\partial x}\Delta x$$

TAKING $\frac{\partial}{\partial x}$ THEN $\frac{\partial F}{\partial x} = -\frac{BA^2}{V}\frac{\partial^2 D}{\partial x^2}\Delta x$

USING $V = A\,\Delta x$ AND EQU'N (2) WE FIND

$$\rho A \frac{\partial^2 D}{\partial t^2} = -\left(-\frac{BA^2}{A\Delta x}\frac{\partial^2 D}{\partial x^2}\Delta x\right)$$

$$\therefore \quad \boxed{\frac{\partial^2 D}{\partial x^2} = \left(\frac{\rho}{B}\right)\frac{\partial^2 D}{\partial t^2}} \Rightarrow v = \sqrt{\frac{B}{\rho}}$$

THE WAVE EQU'N FOR $D = D(x,t)$! SOUND IS A WAVE THAT TRAVELS AT $v = \sqrt{\frac{B}{\rho}}$.

NOTE

- SOL'NS SAME AS BEFORE
- SUPERPOSITION "
- ENERGY "