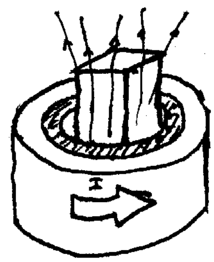
PHYS 195: WEEK 12 SOLUTIONS

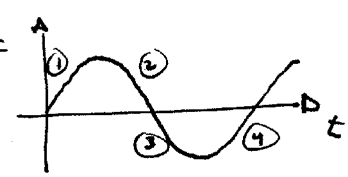
(1.) THOMSON JUMPING RING! FIRST, HOW CAN WE GET ANY LIFT?

SUPPOSE THE CURRENT IS AS SHOWN AND IS INCREASING.



THEN THERE IS A DIVERGING B FIELD AS SHOWN – AND IT IS INCREASING. THE INDUCED CURRENT WILL OPPOSE THE CHANGE IN FLUX SO THE INDUCED CURRENT WILL BE IN THE OPPOSITE DIRECTION (CLOCKWISE). HENCE, THIS IS THE SAME CASE AS WEEK 11 NUMBER 4 AND THE CONDUCTING RING LIFTS OFF THE BASE! [BTW, THE FORCE THAT DOES THIS WORK IS ELECTRICAL.] THIS IS 70% OF THE SOLUTION.

THE CURRENT "ALTERNATES":



I JUST DISCUSSED ①. AS FOR THE REST:

②  INDUCED → SUPPORTS THE B-FIELD

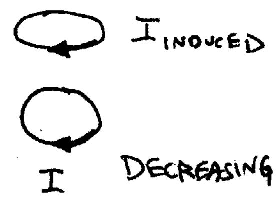
I DECREASING ⇒ FLUX IS DECREASING ⇒ CURRENT INDUCED

SO THE RING IS ATTRACTED! BUT IT HAS ALREADY JUMPED UP AND THE FIELD IS WEAKER, SO THE |FORCE| IS SMALLER.

③  I INDUCED ⇒ RING JUMPS! LIKE ①

I INCREASING

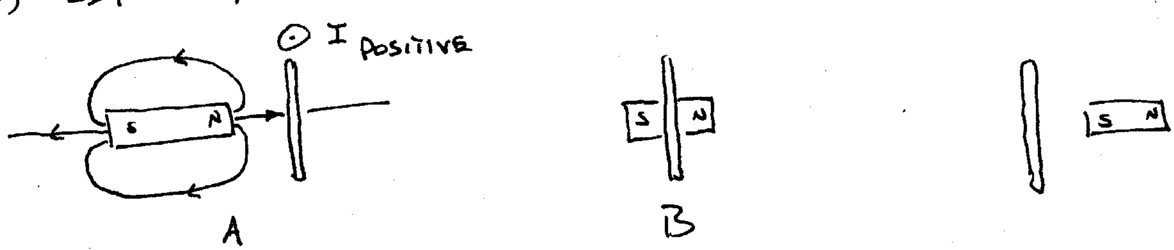
(4)



=> LOOP IS ATTRACTED AS IN (2)

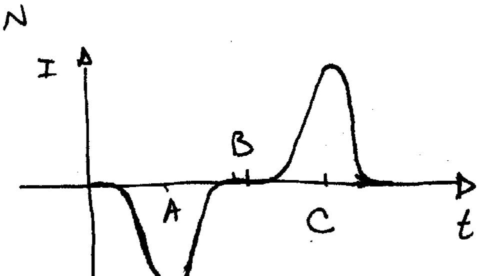
HMM, FOR A FIXED RING THE TIME-AVERAGED FORCE VANISHES! BUT IF WE START AT $t=0$ AS ABOVE FOR EXAMPLE, THE JUMP WILL JUMP, ACCELERATING UPWARDS. BY THE TIME THE RING GETS TO (2) THE MAGNETIC FIELD IS WEAKER SO THE ATTRACTING FORCE IS LOWER. SO THE RING CONTINUES TO JUMP (BUT HAS DOWNWARD ACCELERATION). IF THE CYCLE BEGINS AT (2) OR (4) THE RING IS INITIALLY PUSHED DOWNWARD. WE DON'T SEE THIS SINCE THE NORMAL FORCE OF THE WOOD SUPPORT PREVENTS THE RING FROM MOVING. THE RING WOULD START TO JUMP AS SOON AS THE CURRENT IN THE COIL INCREASES (IN EITHER DIRECTION).

(2.) LET POSITIONS "A, B AND C" BE

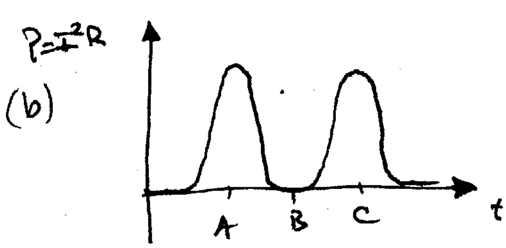


THEN

(a)

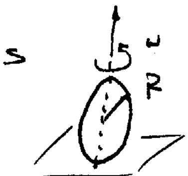


(b)



THE PRECISE SHAPE DEPENDS ON HOW B VARIES ALONG THE MAGNET'S LENGTH

(3.) AS THE QUARTER SPINS THE MAGNETIC FLUX CHANGES. FARADAY TELLS US THAT A E-FIELD IS INDUCED.



$$\oint E \cdot dl = -\frac{d}{dt}\int B \cdot A = -\frac{d}{dt} BA\cos\theta = -\frac{d}{dt} BA\cos(\omega t) \qquad \underline{R \simeq 1.4\,cm}$$

$$= BA\omega \sin(\omega t)$$

WHERE ω IS THE ANGULAR VELOCITY, A THE AREA (πR^2) AND B IS THE MAGNITUDE OF THE EARTH'S MAGNETIC FIELD.

$$\left[\omega = \frac{200\ \text{REV}}{\text{min}} \cdot \frac{1\ \text{min}}{60\ \text{s}} \cdot \frac{2\pi\ \text{rad}}{1\ \text{REV}} \simeq 21\ \text{rad s}^{-1} \right]$$

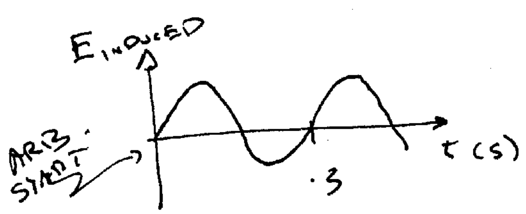
THE LHS IS

$$\oint E \cdot dl = E \cdot 2\pi R$$

So

$$E = \frac{B\cancel{\pi} R^{\cancel{2}}\omega \sin(\omega t)}{2\cancel{\pi}\cancel{R}} \simeq \frac{(10^{-4})(.014\,m)(21\ \text{rad s}^{-1})\sin(21t)}{2}$$

$$\simeq \left(1.5 \times 10^{-5}\ \frac{N}{C}\right)\sin(21t)$$



(4.) (a) THE LOOP REACHES TERMINAL SPEED SINCE THE INCREASING FLUX INDUCES A CURRENT THAT CONSUMES ENERGY AT A RATE OF I^2R (HEAT IN THE RESISTOR).

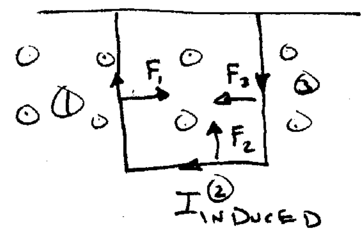
(b.) $\mathcal{E} = \oint E \cdot dl = -\frac{d}{dt}\int B \cdot A = -Bwv$ (SINCE AREA INCREASES WITH V)

SINCE $\mathcal{E} = IR$ THEN

[(c.)] $I = \frac{-Bwv}{R}$. THIS THE INDUCED CURRENT. IT IS CLOCKWISE

THE LOOP HAS MAGNETIC FORCES SO LOOKING

AT $\vec{F} = I\vec{\ell} \times \vec{B}$:



SO $|F_1| = |F_3|$. ~~SO~~ THERE'S NO OVERALL HORIZONTAL FORCE.

$$F_2 = IWB = \frac{B^2 W^2 V}{R}$$

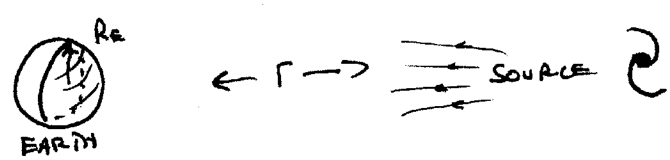
FOR TERMINAL VELOCITY $(a_y = 0)$ V_T

$$F_g - F_B = 0 \quad \Rightarrow \quad mg = \frac{B^2 W^2 V_T}{R}$$

$$\Rightarrow \quad \boxed{V_T = \frac{mgR}{B^2 W^2}}$$

(d.) IT WOULD FALL AS USUAL, $a = g$, SINCE THERE WOULD BE NO CHANGE IN MAGNETIC FLUX.

(5.) A PICOWATT (10^{-12} W) FOR "ALL OF EARTH'S SURFACE". BY THIS I'LL ASSUME THE AREA OF THE DISK INTERUPTING THE SIGNAL.*



SO THE INTENSITY WOULD BE $I = \frac{P}{A} = \frac{P}{\pi R_E^2}$

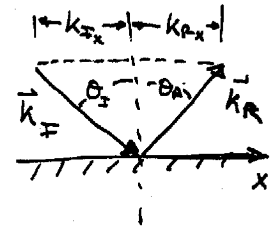
THE TELESCOPE HAS A RADIUS OF $R_T = 150$ m SO IT WOULD RECEIVE

$$P = I \pi R_T^2 = P \frac{R_T^2}{R_E^2} \simeq 10^{-12} \frac{(150\text{m})^2}{(6400\text{km})^2} \simeq 5 \times 10^{-22}\ \text{W}\ (!)$$

(7.) FROM THE LECTURE NOTES SINCE ALL THE BOUNDARY CONDITIONS ON $\vec{E}$ ARE OF THE FORM

$$\vec{E}_{I_0} \cos(\vec{k}_I \cdot \vec{r} - \omega t) + \vec{E}_{R_0} \cos(\vec{k}_R \cdot \vec{r} - \omega t) = \vec{E}_{T_0} \cos(\vec{k}_T \cdot \vec{r} - \omega t)$$

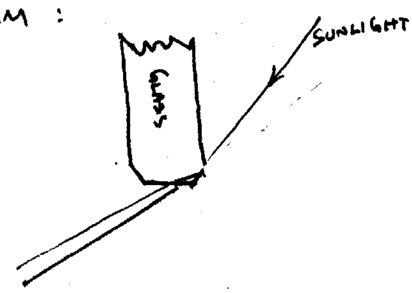
AT $z=0$. THE ARGUMENTS OF THE COSINES MUST BE THE SAME - "THEY ALL WIGGLE TOGETHER". HENCE, FOR $y=0$



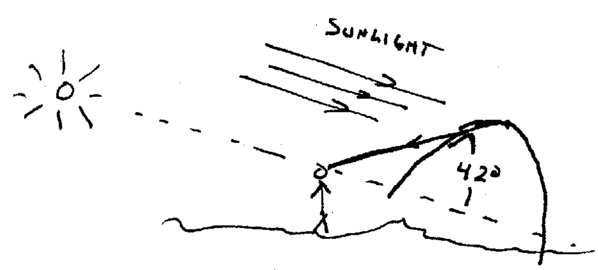
$$k_{I_x} = k_{R_x}$$

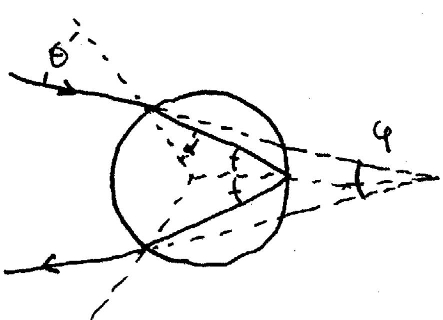
AND SO $\theta_I = \theta_R$ SINCE $|k_I| = |k_R| = \frac{n_1 \omega}{c}$

(8) THE DEPARTMENT SIGN HAS A GLASS FRONT. THE BEVEL ON THE OUTSIDE EDGE CREATES A PRISIM :



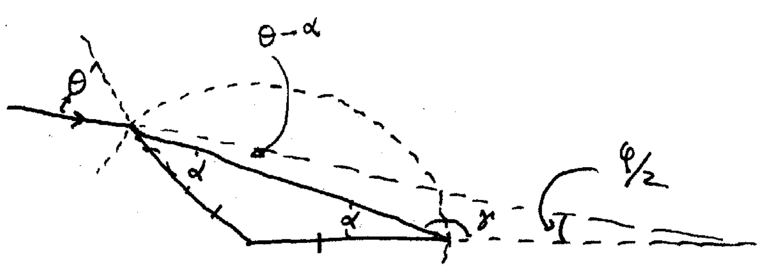
(9.) RAINBOW I : FINDING THE ANGLE 42° ("THE ANSWER TO LIFE, THE UNIVERSE AND EVERYTHING.")





GEOMETRY FROM CLASS. BY SNELL'S, $\sin\alpha = \frac{\sin\theta}{n}$

WE HAVE



SO $(\theta - \alpha) + \gamma + \frac{\varphi}{2} = \pi$

BUT $\gamma = \pi - \alpha$ SO $\frac{\varphi}{2} = 2\alpha - \theta$ HENCE

$$\varphi = 4 \arcsin\left(\frac{\sin\theta}{n}\right) - 2\theta$$

(b.) THE MAXIMUM MAY BE FOUND VIA $\frac{d\varphi}{d\theta} = 0$.

$$\frac{d\varphi}{d\theta} = 4\left(\frac{1}{\sqrt{1 - \frac{\sin^2\theta}{n^2}}}\right)\left(\frac{\cos\theta}{n}\right) - 2 = 0$$

$$\Rightarrow \frac{\cos\theta}{n\sqrt{1 - \frac{\sin^2\theta}{n^2}}} = \frac{1}{2}.$$ SQUARING GIVES

$$\frac{\cos^2\theta}{n^2} = \frac{1}{4}\left(1 - \frac{\sin^2\theta}{n^2}\right).$$ WITH $\sin^2\theta = 1 - \cos^2\theta$

$$\Rightarrow \frac{3}{4}\cos^2\theta = \frac{n^2}{4} - \frac{1}{4} \Rightarrow \theta_{max} = \arccos\left[\sqrt{\frac{1}{3}(n^2-1)}\right]$$

FOR H_2O, $n \cong 1.333$, $\theta_{max} \simeq 59°$ AND

$$\varphi_{MAX} = 4 \arcsin\left(\frac{\sin\theta_{max}}{n}\right) - 2\theta_{max} \simeq 42°$$

(c.) φ IS AT A MAXIMUM AT $\theta \simeq 59°$. IT IS STATIONARY AND ~~THIS~~ RELATIVELY FLAT AT THIS ANGLE. HENCE AS YOU VARY THE INCIDENT ANGLE θ, φ DOESN'T VARY MUCH. SO MORE LIGHT IS DEFLECTED AT THIS ANGLE OF $\varphi \simeq 42°$. "MORE RAYS $\Rightarrow$ MORE LIGHT $\Rightarrow$ BRIGHT!"

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In[3]:= Plot[4 * ArcSin[Sin[Theta] / 1.33] - 2 * Theta, {Theta, 0, 0.8 * Pi}]
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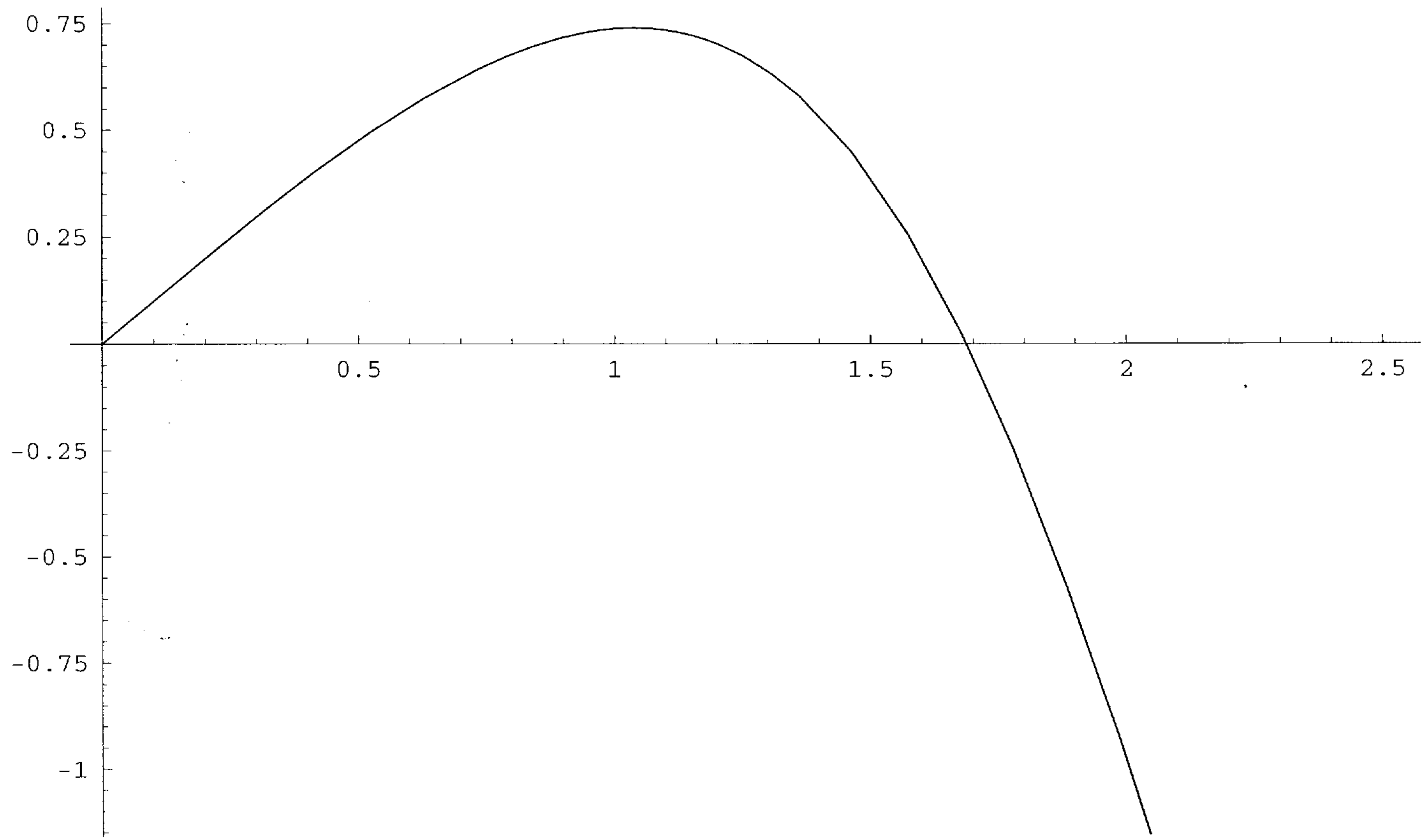


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Out[3]= - Graphics -
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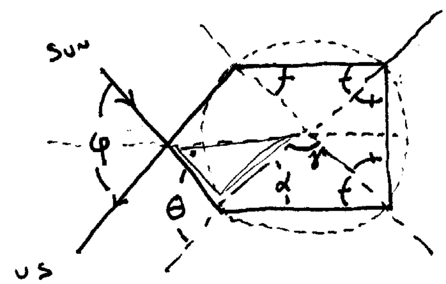
(11) (BONUS 1 PT.) MORE RAINBOW GEOMETRY

SECONDARY RAINBOWS HAVE TWO INTERNAL REFLECTIONS



BY SNELL'S RELATION $\sin\theta = n\sin\alpha$ OR $\alpha = \arcsin\left(\frac{\sin\theta}{n}\right)$

FROM THE ABOVE GEOMETRY,

$$\gamma = \pi - 2\alpha$$

AND FROM THE PINK TRIANGLE

$$\frac{\varphi}{2} + (\pi - \theta) + \frac{2\pi - 3\overbrace{(\pi - 2\alpha)}^{\gamma}}{2} = \pi$$

OR, AFTER SOME ALGEBRA ...

$$\varphi = \pi - 6\alpha + 2\theta$$

$$= \pi - 6\arcsin\left(\frac{\sin\theta}{n}\right) + 2\theta \qquad (**)$$

TAKING THE DERIVATIVE AS IN (5)

$$\frac{d\varphi}{d\theta} = \frac{-3}{\sqrt{1 - \frac{\sin^2\theta}{n^2}}}\left(\frac{\cos\theta}{n}\right) + 1 = 0$$ ... DOING MORE ... ALGEBRA

$$\Rightarrow \cos\theta = \sqrt{\frac{n^2-1}{8}} \Rightarrow \theta \simeq 73.9^\circ$$

PLUGGING THIS RESULT BACK INTO THE EQ'N FOR THE DEFLECTION ANGLE φ, (**)

$$\varphi_{EXTREMA} \cong \underline{\underline{50.3^{\circ}}}$$

I USED $n = 1.33$ THROUGHOUT.

(12.) BONUS — TURN IN TO SETH.