PHYS 195 Week 1 Solutions Spring 2009

(1) SIG FIGS

(2)(a) Too many sig figs in the best estimate $x = 3 \pm 1$ mm
(b) silly number of sig figs, more clearly written in scientific notation $t = 1.23 \pm 0.05$ x 10^6 s
(c) too many sig figs in uncertainty 5.33 ± 0.03 x 10^(-7) m
(d) scientific notation, rounding $r = 5.4 \pm 0.3$ x 10^(-10) m or 540 ± 30 nm

(3) THE DISPLACEMENT AND VELOCITY VECTORS ARE IN THE SAME DIRECTION EVERY TIME THE SYSTEM IS MOVING AWAY FROM EQUILIBRIUM. DISPLACEMENT AND ACCELERATION ARE NEVER IN THE SAME DIRECTION SINCE

$$a = \frac{d^2x}{dt^2} = -\omega^2 x$$ MEANS $\vec{a}$ IS OPPOSITE $\vec{x}$.

(4)

AS IN (4) $k = \frac{mg}{Ax}$ SO $\omega = \sqrt{\frac{mg}{Ax(m+M)}}$

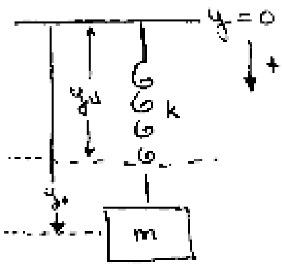
WHERE m IS THE MASS OF THE PERSON AND M IS THE MASS OF THE CAR. HENCE

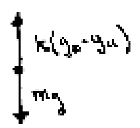
$$f = \left(\frac{1}{2\pi}\right)\sqrt{\frac{mg}{(m+M)Ax}} \simeq \underline{1\ \text{Hz}}$$

You can also include explicitly the factors of 4 for the different springs. The frequency, though, will still be about 1 Hz.

(5) (a) Hooray!
(b) I found the red mass was about 300 g
(c) T = 0.52 s
(d) Given their choice for the zero of gravitational potential energy, the quantity of kinetic energy is much smaller than the potential energy.
(e) Oddly enough, it is a bit below what appears to be the “ground”.
(f) The amplitude decays. The spring and air are heating up.

(6)





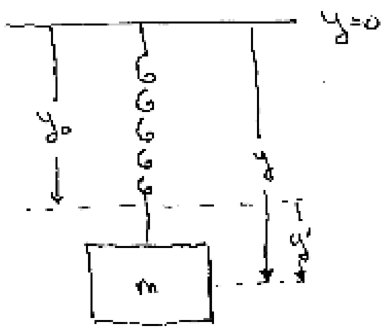
• EQUILIBRIUM: $\Sigma F = 0$

$$+mg - k(y_0 - y_u) = 0$$

$$\Rightarrow \quad mg - ky_0 + ky_u = 0$$

$$\therefore \quad \boxed{y_0 = \frac{mg}{k} + y_u} \qquad (1)$$

• MOTION AROUND EQUILIBRIUM, LET'S CHOOSE THE SAME ZERO PT. FOR y.



. NEWTON'S 2ND GIVES

$$+mg - k(y - y_u) = m\frac{d^2y}{dt^2}$$

NOW, $y = y_0 + y'$ ← DISPLACEMENT FROM EQUILIBRIUM POSITION

HENCE,

$$mg - k(y_0 + y' - y_u) = m\frac{d^2y}{dt^2}$$

FROM EQUN (1) WE HAVE $\cancel{mg - ky_0 + ky_u} - ky' = m\frac{d^2y}{dt^2}$

SO THAT $\quad -ky' = m\frac{d^2y}{dt^2}$

NOW $\frac{d^2y'}{dt^2} = \frac{d^2y}{dt^2}$ SO $\boxed{\frac{d^2y'}{dt^2} + \frac{k}{m}y' = 0}$

SHM !

(7)

) WE'RE GIVEN THE MASS AND PERIOD. WITH

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$

WE CAN FIND WHAT WE NEED. FIRST, $\omega = \frac{2\pi}{T} \simeq 11\,s^{-1}$

(a) FROM THE INITIAL CONDITIONS $(y(0) = 0.1\text{ m})$.

$$y(t) = 0.1\cos(11t)\text{ m}$$

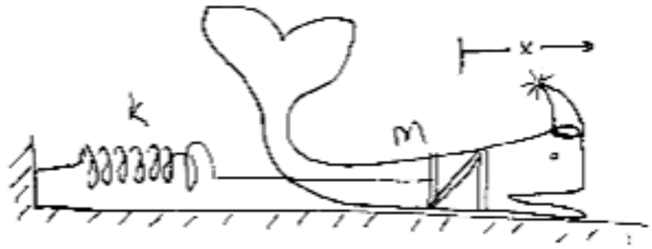
(b.) $\frac{T}{4} \simeq 0.14\text{ s}.$

(c.) $v_{max} = A\omega \cong 1.14\,\frac{m}{s} \simeq 1\text{ m/s}$

(d.) $a_{max} = A\omega^2 \simeq 13\text{ m/s}^2 \simeq 10\text{ m/s}^2$ OCCURS WHEN $x = A$, AT THE TURNING POINTS, OR $t=0$ AND $t=T$

(8)

(a.)



IF $x > 0$ THEN $F = -kx$ SO

$$m \frac{d^2x}{dt^2} = -kx \quad \Rightarrow \quad \frac{d^2x}{dt^2} + \frac{k}{m} x = 0 \quad \text{SHM!}$$

(b.) $T = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{m}{k}} \simeq \underline{5.1\,s}$

(c.) THE SOL'N IS $X = A \sin(\omega t + \varphi)$ SO

$X(0) = 0 \Rightarrow A \sin(\varphi) = 0 \Rightarrow \varphi = 0$ AND

$\frac{dx}{dt}(0) = 1.3 \Rightarrow \omega A \cos(0) = 1.3 \Rightarrow A = \frac{1.3}{\omega} \simeq 1.1\,m$

HENCE

$$\underline{X = 1.1 \sin(1.23\,t)\ m}$$

(9) SINCE $F = -kx$ AND $U(x) = -\int F\,dx$ WE HAVE

$$U(x) = +\int kx\,dx = \frac{1}{2}kx^2 + C$$

WE USUALLY SET $C = 0$