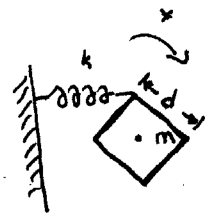
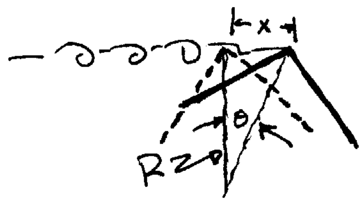
PHYS 195: WEEK 3 SOLUTIONS

(1.) THE CUBE ROTATES SO WE SHOULD USE TORQUE. TO RELATE ~~THESE~~ TORQUE TO THE RESTORING FORCE I'LL THINK ABOUT DISPLACING THE CUBE A LITTLE THIS WAY * :

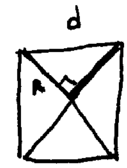


$$\begin{cases} m = 3.00\,kg \\ d = 6.00\,cm \\ k = 1200\,N/m \end{cases}$$



GEOMETRY:

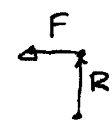
$d^2 = 2R^2$



FOR SMALL ANGLES!

SO $x \cong R\theta = \frac{d}{\sqrt{2}}\theta$. THEN THE FORCE IS

$F = -kx = -\frac{kd\theta}{\sqrt{2}}$. THE FBD IS



GIVING $\tau = r \times F = RF = -\frac{kd\theta}{\sqrt{2}} \cdot \frac{d}{\sqrt{2}} = -\frac{kd^2\theta}{2}$, A RESTORING TORQUE

NOW $\tau = I\alpha \Rightarrow -\frac{kd^2\theta}{2} = I\frac{d^2\theta}{dt^2}$ OR

$$\frac{d^2\theta}{dt^2} + \frac{kd^2}{2I}\theta = 0$$

HRW GIVES AN I OF $\frac{1}{12}m(2d^2)$ (ON PAGE 253)

OR $I = \frac{1}{6}md^2$ SO, THE EOM IS

$$\frac{d^2\theta}{dt^2} + \frac{k\cancel{d^2}}{\cancel{2}\frac{1}{\cancel{6}_3}m\cancel{d^2}}\theta = 0 \Rightarrow \omega_0 = \sqrt{\frac{3k}{m}}$$

HENCE $T = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{m}{3k}} \simeq \frac{2\pi}{\sqrt{1200}}\,s = \underline{0.18\,s}$

(2.) MANY ANSWERS ... I HAVE A TREE OUTSIDE MY OFFICE WINDOW THAT IS OCCASIONALLY BUFFETED BY APPARENTLY PERIODIC BURSTS OF WIND. THE AMPLITUDE OF SWAYING CAN BE SURPRISINGLY LARGE.

(3.) (a.) WHEN THE TOWER OSCILLATES IT WILL DRIVE THE MASS-ON-A-SPRING. THIS REDUCES THE ENERGY STORED IN THE OSCILLATIONS OF THE BUILDING, THUS REDUCING THE AMPLITUDE OF THE OSCILLATION.

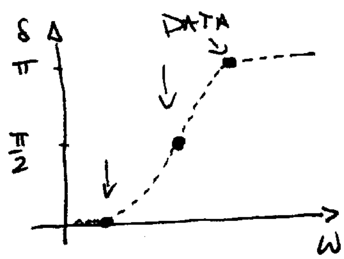
(b.) SINCE $A(t) = A_0 e^{-\alpha t}$ WE HAVE

$$\begin{cases} A(t_0) = 1.4\text{m} = A_0 e^{-\alpha t_0} \\ A(t_0+10T) = A_0 e^{-\alpha(t_0+10T)} = 0.80\text{m} \end{cases} \Rightarrow \frac{A(t_0)}{A(t_0+10T)} = \frac{1.4}{.8} = e^{+\alpha 10T}$$

$$\Rightarrow \alpha = \frac{\ln(1.4/.8)}{10T} \Rightarrow b = \frac{2m\ln(1.75)}{10T} \simeq \underline{6700\ \tfrac{\text{kg}}{\text{s}}}$$

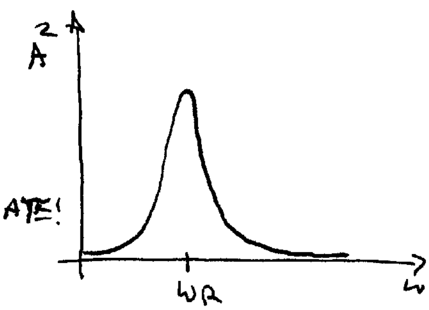
(4.)(a) USING A MUG AND 3 RUBBER BANDS, I FIND A PERIOD OF ABOUT 1s (RESULTS WILL VARY!). SO $\omega_0 = \frac{2\pi}{T} \simeq \underline{6\ s^{-1}}$.

(b) AFTER OBSERVATION, AND USING OUR CONVENTIONS FOR δ I FIND:



THE FIRST TWO PTS ARE RELATIVELY EASY TO SEE. THE LAST IS TRICKY!

(5.) WE'RE LOOKING FOR ω_R: THE LOCATION OF THE PEAK IN A^2. SO LET'S DIFFERENTIATE!



$$\frac{dA^2}{d\omega} \propto \cancel{2}(\omega_0^2-\omega^2)(-2\omega) + \frac{\cancel{2}\omega b^2}{m^2} = 0 \quad \text{AT MAX}$$

$$\Rightarrow \quad \omega_R^2 = \omega_0^2 - \frac{b^2}{2m^2} \quad \Rightarrow \quad \omega_R = \sqrt{\omega_0^2 - \frac{b^2}{2m^2}}$$

(b.) FOR LIGHTLY DAMPED $\left(\frac{b}{m\omega_0} \ll 1\right)$ SYSTEMS

$$\omega_R = \omega_0\left[1 - \frac{b^2}{2\omega_0^2 m^2}\right]^{1/2} \simeq \omega_0\left(1 - \frac{b^2}{4\omega_0^2 m^2}\right) \simeq \omega_0$$

USING $(1+x)^n \simeq 1 + nx$

SO ω_R (AND ω_d) ARE APPROXIMATELY EQUAL TO ω_0, THE NATURAL ANGULAR FREQUENCY.

(c.) AS WE SAW IN CLASS,

$$Q = \frac{\text{ENERGY STORED IN OSCILLATOR}}{\text{ENERGY DISSIPATED PER RADIAN}}$$

THE ENERGY STORED IS $E = \frac{1}{2} m\omega^2 A^2 = \frac{1}{2} m \frac{g}{\ell} \ell^2 \theta_0^2$

$$= \frac{1}{2} m g \ell \theta_0^2$$

WHERE "m" IS THE MASS OF THE PENDULUM, AND ℓ IS ITS LENGTH. NUMERICALLY $E \cong$

THE ENERGY DISSIPATED IS EQUAL TO THE ENERGY ADDED BY THE DRIVING MECHANISM. THUS, AS THE WEIGHT FALLS, $m_2 g h$ (WHERE m_2 IS THE MASS OF THE WEIGHT) IS DISSIPATED OVER 24 HOURS.

THE "PER RADIAN" IS A TIME OF $1/\omega$ SO,

THE ENERGY STORED IS (IN SLIGHTLY DIFFERENT NOTATION)

$$E = \frac{1}{2} m \omega^2 \ell^2 \theta_0^2$$

WHILE THE ENERGY DISSIPATED IN $T = 24$ HRS IS

$$\frac{\Delta E}{\Delta T} = \frac{m_2 g h}{T}$$

HENCE "ENERGY DISSIPATED PER RADIAN" IS

$$\left|\frac{dE}{dt}\right| \frac{1}{\omega} \simeq \frac{\Delta E}{T} \frac{1}{\omega} = \frac{m_2 g h}{T \omega}$$

$$\Rightarrow \quad Q = \frac{\frac{1}{2} m \omega^2 \ell^2 \theta_0^2}{\frac{m_2 g h}{T\omega}} = \frac{m \omega^3 \ell^2 \theta_0^2 T}{m_2 g h} = \frac{m g^{3/2} \ell^2 \theta_0^2 T}{m_2 g \ell^{3/2} h}$$

$$= \frac{m \sqrt{\ell g}\, \theta_0^2 T}{m_2 h} \simeq \underline{\underline{2.6}}.$$

(7.) I'LL ESTIMATE AN INITIAL AMPLITUDE OF 20 CM AND A FINAL AMPLITUDE SO SMALL IT LOOKS AT REST, ABOUT 0.5 CM. OBVIOUSLY YOUR NUMBERS MAY BE DIFFERENT! NOW

$$A(t) = X_m e^{-\alpha t} = X_m e^{-\frac{b}{2m}t} = X_m e^{-\frac{\cancel{m}\omega t}{2\cancel{m}Q}}$$

SINCE $Q = \frac{m\omega}{b}$ FOR LIGHTLY DAMPED SYSTEMS

FOR THE TIME INTERVAL $t = 4$ HRS,

$$0.5\,\text{cm} \sim 20\,\text{cm}\, e^{-\omega t/2Q} \quad \Rightarrow \quad Q = \sqrt{\frac{g}{\ell}}\left(\frac{t}{2\ln(20/.5)}\right)$$

$$\simeq 1400$$