PHYS 195: WEEK 4 SOLUTIONS

(1) WE'LL PLOT A^2 IN TERMS OF (F_0/m) AND ω_{DRIVE} IN TERMS OF ω_0. WITH THIS IN MIND WE HAVE

$$\frac{A^2}{(F_0/m)} = \frac{1}{(\omega_0^2-\omega^2)^2 + \left(\frac{b\omega}{m}\right)^2} = \frac{1}{\omega_0^4\left[\left(1-\left(\frac{\omega}{\omega_0}\right)^2\right)^2 + \left(\frac{b\omega}{m\omega_0^2}\right)^2\right]}$$

$$= \omega_0^{-4}\left[\left(1-\left(\frac{\omega}{\omega_0}\right)^2\right)^2 + \left(\frac{\omega}{\omega_0}\frac{b}{m\omega_0}\right)^2\right]^{-1}$$

$$\simeq \omega_0^{-4}\left[\left(1-\left(\frac{\omega}{\omega_0}\right)^2\right)^2 + \left(\frac{\omega}{\omega_0}\frac{1}{Q}\right)^2\right]^{-1}$$

I'LL PLOT ~~WHAT IS AN~~ THE $[\cdots]^{-1}$ BIT FOR $Q = 35$. SO ON THE VERTICAL I'LL HAVE THE DIMENSIONLESS MEASURE OF AMPLITUDE, $\left(\frac{A\omega_0^2}{F_0/m}\right)^2$. SEE THE NEXT PAGE.

(2.) (a) $\frac{\partial z}{\partial x} = 3x^2y + ye^{xy}$ (b) $\frac{\partial z}{\partial y} = x^3 + xe^{xy}$

(c.) $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x} = 3x^2 + e^{xy} + xye^{xy}$ (MIXED PARTIALS ARE EQUAL, THE CHANGES IN z W/R TO x AND y ARE INDEPENDENT.

(d.) $\frac{\partial^2 z}{\partial x^2} = 6xy + y^2e^{xy}$.

(3.) FOR LIGHTLY DAMPED SYSTEMS $\omega_0 \simeq \omega_d$ AND $Q \simeq \frac{m\omega_0}{b}$. SO USING THE PHASE DERIVED IN

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> plot(1/((1-omega^2)^2+omega^2/35^2), omega=0..2);
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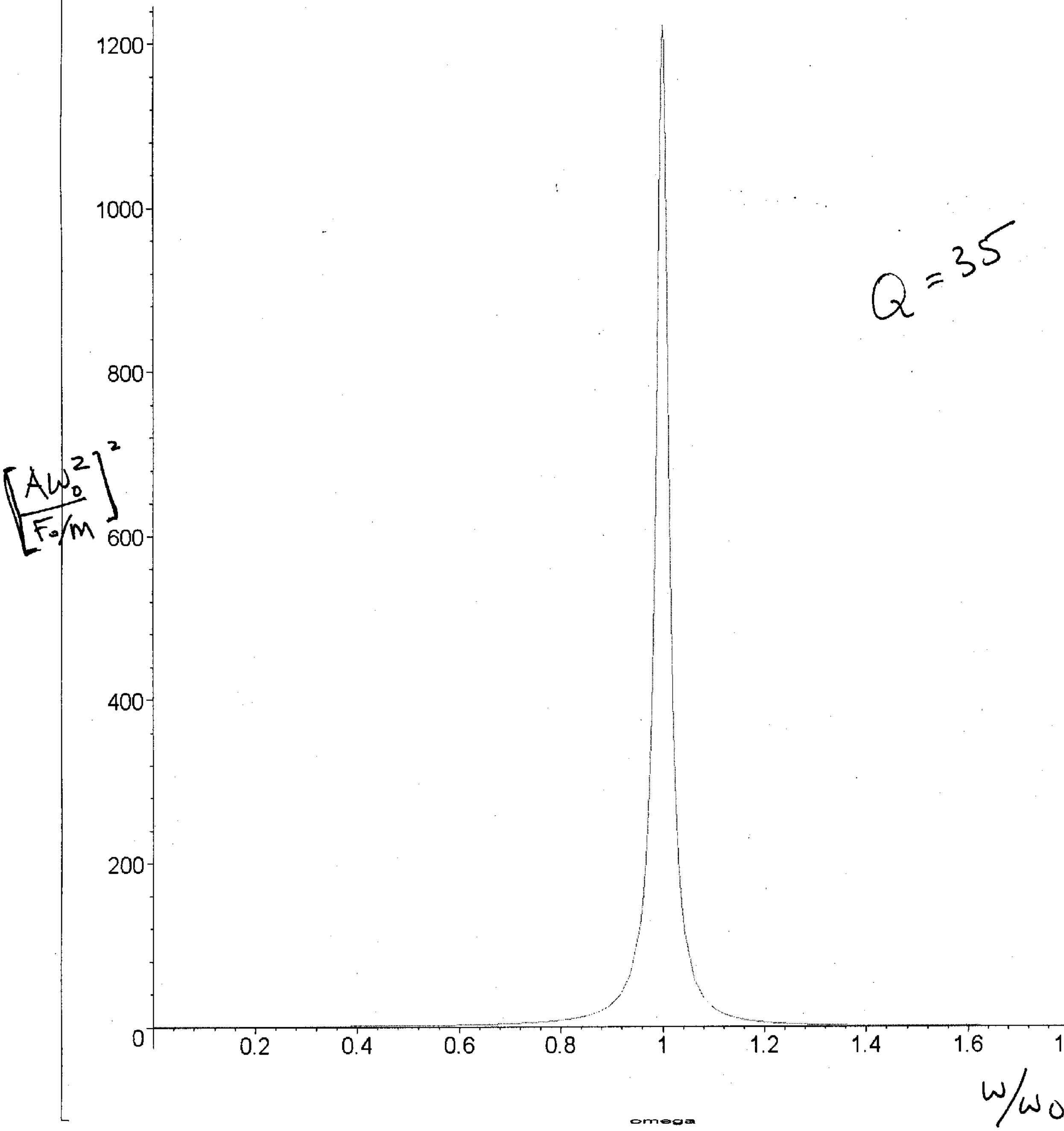


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> plot(1/((1-omega^2)^2+omega^2/3^2), omega=0..2);
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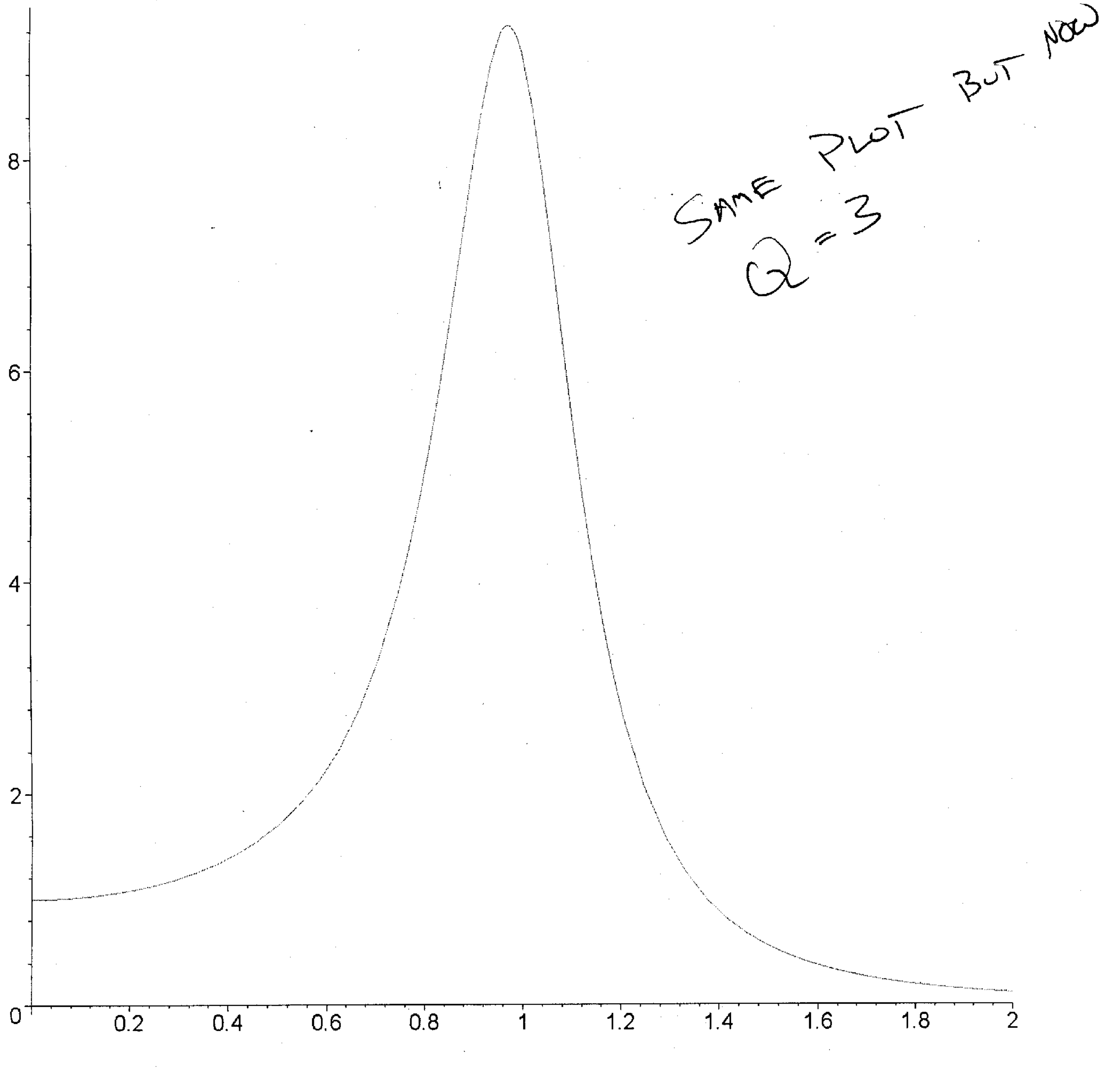


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CLASS,

$$\tan\delta = \frac{b\omega}{m(\omega_0^2-\omega^2)} = \frac{b\omega}{m\omega_0^2\left(1-\frac{\omega^2}{\omega_0^2}\right)} \cong \frac{\omega}{Q\omega_0\left(1-\frac{\omega^2}{\omega_0^2}\right)}$$

$$= \left(\frac{1}{50}\right)\left(\frac{9}{7.7}\right)\left[1-\left(\frac{9}{7.7}\right)^2\right]^{-1} \quad \therefore \quad \text{"}\delta\text{"} \simeq -0.06 \text{ rad}$$

HOWEVER, THE PHASE SHIFT δ IS BETWEEN 0 AND π AND, FURTHERMORE, INCREASES FROM 0, THROUGH $\frac{\pi}{2}$ AT RESONANCE, TO π FOR $\omega \gg \omega_0$. AS YOU CAN SEE FROM THE PLOT, WE SHOULD SHIFT THIS UP BY π. HENCE $\underline{\underline{\delta \simeq 3.08 \text{ rad} \simeq 3 \text{ rad}}}$. THE CONFUSION COMES FROM THE CHANGE OF SIGN OF ARCTAN AT RESONANCE.

(4.) (a.) FOR "FIXED END" THE WAVE REFLECTS INVERTED; IT ENJOYS A PHASE SHIFT BY π. FOR "LOOSE END" THE WAVE REFLECTS UPRIGHT WITH NO PHASE SHIFT. FOR "NO END" THE WAVE DOES NOT REFLECT.

(b.) THE AMPLITUDE INCREASES! THE DEFAULT SETTING IS AT A <u>RESONANT</u> FREQUENCY.

(5.) TRANSVERSE AND TORSIONAL BUT NO, LONGITUDINAL!

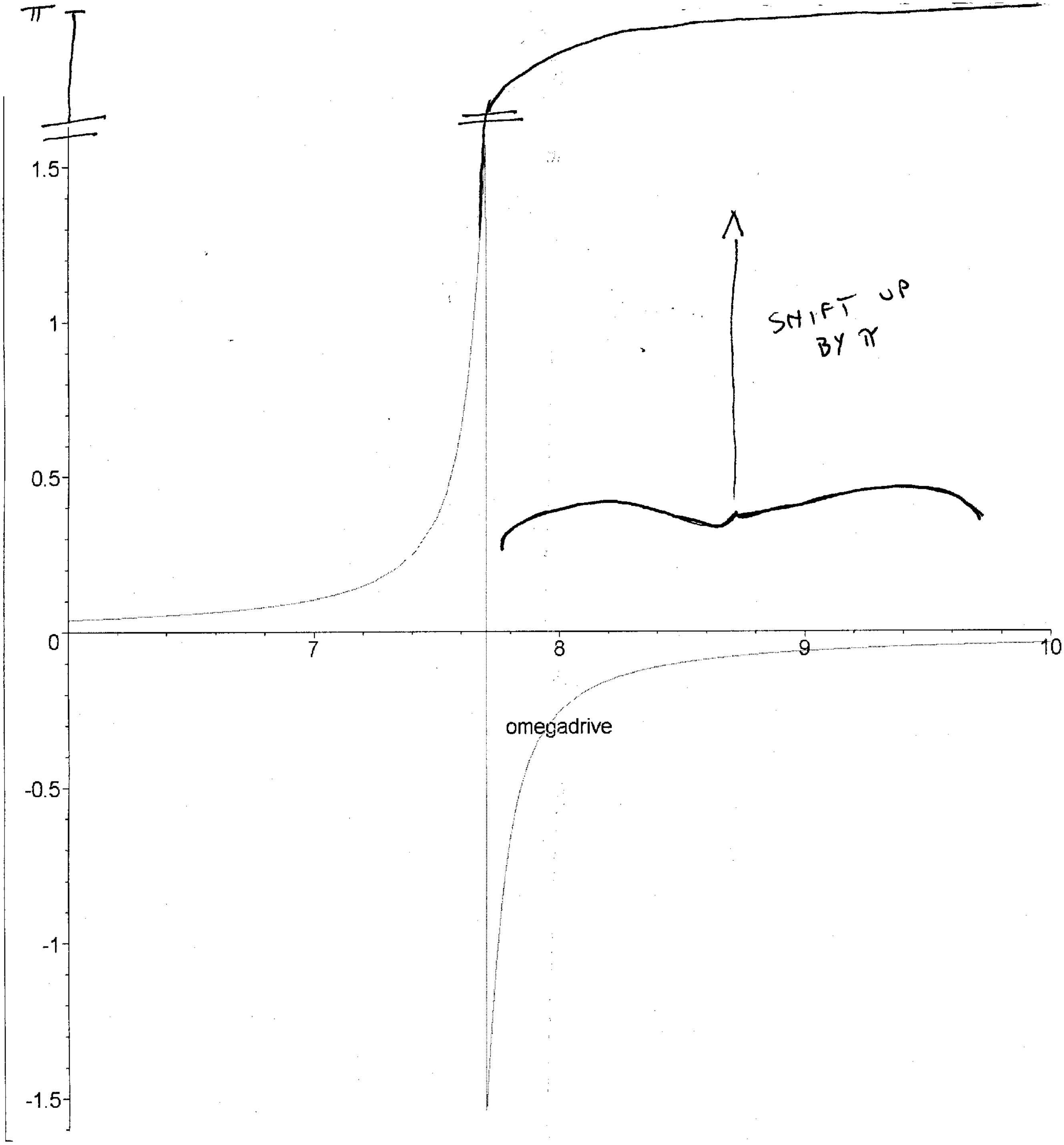
(6.) IF $y(x,t) = 0.050 \sin(6.00x + 12.0t)$ THEN

(a.) THE WAVE IS LEFT MOVING

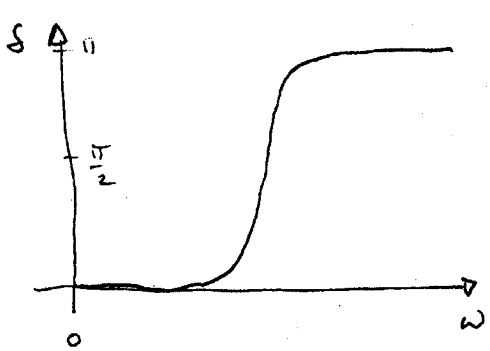
(b.) $k = 6.00 \text{ m}^{-1} \Rightarrow \lambda = \frac{2\pi}{k} \simeq 1.05 \text{ m}$

$\omega = 12.0 \text{ s}^{-1} \Rightarrow f = \frac{\omega}{2\pi} \simeq 1.91 \text{ Hz}$

$\Rightarrow V = \lambda f = \frac{\omega}{k} = \underline{\underline{2.00 \text{ m/s}}}$


π
1.5
1
0.5
0
-0.5
-1
-1.5
7
8
9
10
omegadrive
SHIFT UP
BY π

[>


δ
π
π/2
0
ω

(c) THE STRING SPEEDS AS IT UNDERGOES SHM ARE GIVEN BY

$$\frac{\partial y}{\partial t} = (-12.0)(0.050)\cos(6.00x + 12.0t)$$

WELL THOSE ARE VELOCITIES SO THE SPEED IS $\left|\frac{\partial y}{\partial t}\right|$.

THE MAX IS $(12.0)(0.050) = 0.6$ m/s AND THE MINIMUM IS 0.

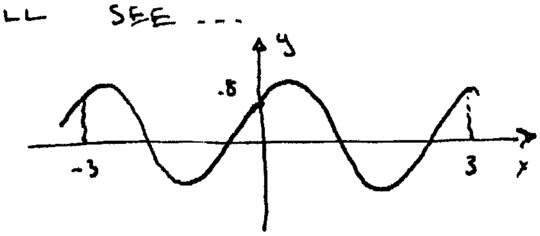
(d.) DELAYED TO NEXT WEEK.

(7.) WE WILL BE LOOKING FOR A SOL'N OF THE TYPE

$$y(x,t) = y_m \sin(kx + \omega t + \varphi)$$

LEFT MOVING!

(a.) AS WE'LL SEE ...



(b.) $\begin{cases} k = \frac{2\pi}{\lambda} = \frac{2\pi}{3} \simeq 2.09 \text{ cm}^{-1} \\ \omega = 2\pi f \simeq 1.26 \times 10^3 \text{ s}^{-1} \\ y_m = 1.00 \text{ cm} \end{cases}$

AT $x = t = 0$, $y = 0.8 = \sin(\varphi) \Rightarrow \varphi = 0.927$ rad (ABOUT 53°)

IS IT MOVING UPWARD?

$$\left.\frac{\partial y}{\partial t}\right|_{x=0} > 0?,$$

CALCULATING, $\frac{\partial y}{\partial t} = y_m \omega \cos(\varphi) > 0$ ✓ YES.

So $y(x,t) = 1.00 \sin\left(2.09\, x + 1.26 \times 10^{3} t + 0.927\right)$ cm

(8.) GIVEN $f = f(\underbrace{kx \pm \omega t}_{z})$ THEN

$$\begin{cases} \dfrac{\partial^2 f}{\partial x^2} = \dfrac{d^2 f}{dz^2}\left(\dfrac{\partial z}{\partial x}\right)^2 = f'' k^2 \\ \dfrac{\partial^2 f}{\partial t^2} = \dfrac{d^2 f}{dz^2}\left(\dfrac{\partial z}{\partial t}\right)^2 = f'' \omega^2 \end{cases}$$

SO THE WAVE EQUIN GIVES

$$\frac{\partial^2 f}{\partial x^2} = \frac{1}{V^2}\frac{\partial^2 f}{\partial t^2} \Rightarrow f'' k^2 = \frac{1}{V^2} f'' \omega^2$$

OR

$$\left(k^2 - \frac{\omega^2}{V^2}\right) f'' = 0 \Rightarrow$$ IF $V = \frac{\omega}{k}$ THEN WE HAVE A SOLIN, AS EXPECTED.