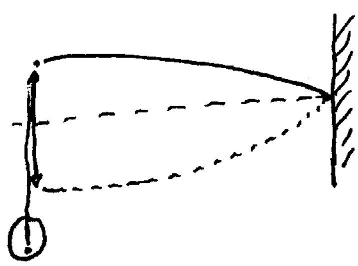
# PHYS 195 WEEK 5 SOLUTIONS

(1.) [ANSWERS WILL VARY DEPENDING ON CARE AND SETTINGS] WITH SETTINGS AMPLITUDE = 11 AND DAMPING = 5 I FOUND STANDING WAVES AT APPROXIMATELY

$$\begin{cases} 32\ \text{Hz}, \text{ WITH } 4 \text{ NODES AND } \lambda = 50\ \text{cm} \\ 64\ \text{Hz} \text{ WITH } 8 \text{ NODES AND } \lambda \simeq 25\ \text{cm} \end{cases}$$

THE FUNDAMENTAL OUGHT TO BE OF THE FORM



WITH $\lambda_1 = 4L$ SO WITH $V = \lambda_1 f_1 = 4L f_1$, THE FUNDAMENTAL SHOULD BE AT $f_1 = \frac{V}{4L}$. HIGHER HARMONICS WOULD BE GIVEN BY $f_n = \frac{(2n-1)V}{4L}$, $n = 1, 2, 3, \ldots$. USING THE FIRST STANDING WAVE CONFIGURATION (AND THE RULER TO FIND $L \simeq 96.5\ \text{cm}$)

$$V = \lambda f = (0.5\ \text{m})(32\ \text{Hz}) \simeq 16\ \text{m/s}$$

$$\Rightarrow f_1 = \frac{V}{4L} \simeq \frac{16}{(4)(0.965)}$$

$$\simeq 4.5(\pm 1)\ \text{Hz}.$$

THE FIRST CONFIGURATION IS $n = 4$ SO $f_4 \simeq 7 f_1 \simeq 32\ \text{Hz}$

THERE IS ANOTHER CONFIGURATION AT $n = 5$, $f_5 \simeq 38\ \text{Hz}$.

(2.) THE AVERAGE RATE OF ENERGY TRANSPORT

IS $\quad \bar{P} = \frac{1}{2}\omega^2 \mu A^2 v$

NOTING $V = \frac{\omega}{k}$ WE HAVE $\bar{P} = \frac{1}{2}\mu\frac{\omega^3 A^2}{k} \simeq \left(\frac{1}{2}\right)(.1)\left(\frac{12^3}{6}\right)(.05)^2$

$$\simeq \underline{\underline{0.036\ \text{J/s}}}$$

(3.) (a) $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{3.5}{0.04}} \simeq 9.4\ \text{rad/s}$

(b) $E_0 = \frac{1}{2}m\omega_0^2 A^2 = \frac{1}{2}kA^2 = \left(\frac{1}{2}\right)(3.5)(0.12)^2 \simeq 0.025\ \text{J}$

(c) ~~THE~~ A GENERAL SOL'N IS GIVEN BY

$$x(t) = A\sin(\omega t + \varphi)$$

SO $\quad x(0) = 0 = A\sin(\varphi) \Rightarrow \varphi = 0 \text{ OR } \pi$

AND $\quad \left.\frac{dx}{dt}\right|_{t=0} = A\omega\cos(\omega t + \varphi) = A\omega\cos(0) > 0$, USING $\varphi = 0$

SINCE THE INITIAL CONDITIONS ARE SATISFIED THE SOL'N IS

$$X(t) = 0.12\sin(9.4t)\ \text{m}$$

(4.) NOW $\frac{\delta T}{T} \simeq 10^{-3}$ AND $\frac{\delta l}{l} \simeq 10^{-3}$. OH WELL IT LOOKS LIKE WE'LL HAVE TO USE BOTH; NEITHER DOMINATES. SO WITH

$$g = \frac{4\pi^2 l}{T^2}$$

$$\delta_\ell g = \left|\frac{\partial g}{\partial \ell}\right| \delta\ell = \frac{4\pi^2}{T^2}\,\delta\ell = g\,\frac{\delta\ell}{\ell}$$

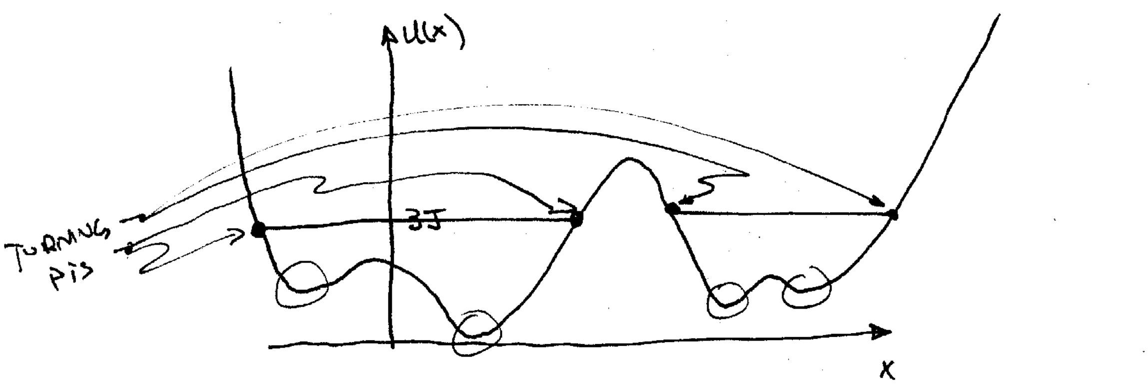
$$\delta_T g = \left|\frac{\partial g}{\partial T}\right| \delta T = 2\,\frac{4\pi^2 \ell}{T^3}\,\delta T = 2\,g\,\frac{\delta T}{T}$$

$$\Rightarrow \quad \delta g = \sqrt{(\delta_\ell g)^2 + (\delta_T g)^2} = g\sqrt{\left(\frac{\delta\ell}{\ell}\right)^2 + 4\left(\frac{\delta T}{T}\right)^2}$$

$$\simeq (9.8198)(0.002) \simeq 9.82 \pm 0.02 \ \mathrm{m/s^2}$$

THIS MAY OR MAY NOT AGREE WITH YOUR MEASUREMENT.

(5.)




(c) NO. THIS IS NOT MOTION AROUND A STABLE EQUILIBRIUM PT.

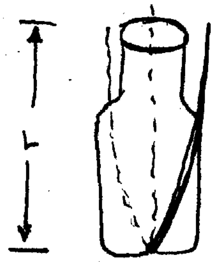
(6.) $X(t) = X_m e^{-\alpha t}\cos(\omega_d t + \varphi)$ WITH $\begin{cases} \alpha = \dfrac{b}{2m} \\ \omega_d = \sqrt{\omega_0^2 - \dfrac{b^2}{4m^2}} \end{cases}$

THIS ~~ODE~~ EOM IS A 2ND ORDER, LINEAR DIFF EQUN WITH CONSTANT COEFFICIENTS

(8.) (a) ANSWERS WILL VARY. I USED A 15 cm BOTTLE.

(b) FOR A BOTTLE $L$ HIGH, THE FUNDAMENTAL MODE IS

(c.)




WITH $\lambda_1 = 4L$. SINCE $V = \lambda f$

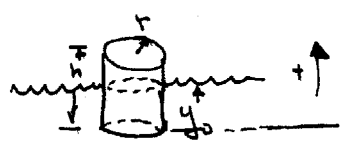
WE HAVE $f_1 = \dfrac{V}{\lambda_1} = \dfrac{V}{4L} \simeq 570\text{ Hz}$.

FOR A $L = 15\text{ cm}$ BOTTLE.

(d.) WITH BOTTLE 1/3 FULL $L \longrightarrow L' = \frac{2}{3}L$

$$\Rightarrow f_1' = \frac{V}{4 \cdot \frac{2L}{3}} = \frac{3}{2} f_1 \simeq 860\text{ Hz}$$ FOR 15 cm BOTTLE.

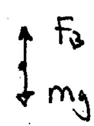
(9.) IN EQUILIBRIUM




$$F_B - mg = 0$$

$$\Rightarrow g \rho_{H_2O} \pi r^2 y_0 = \rho_{Ducky} \pi r^2 h g \qquad (1)$$

DISPLACING THE DUCKY INTO THE $H_2O$ $(y<0)$ GIVES THE FBD




NEWTON F=ma GIVES

$$F_B - mg = m \frac{d^2y}{dt^2}$$

OR SINCE $y < 0$

$$g \rho_{H_2O} \pi r^2 (y_0 - y) - mg = m \frac{d^2y}{dt^2}$$ SO THAT

BY EQU'N (1)

$$\cancelto{0}{g\rho_{H_2O}\pi r^2 y_0 - mg} - g\rho_{H_2O}\pi \cancel{r^2} y = \rho_{Ducky}\,\pi \cancel{r^2} h \frac{d^2y}{dt^2}$$

$$\therefore \quad \frac{d^2y}{dt^2} + \frac{g\rho_{H_2O}}{\rho_{Ducky}\,h}\,y = 0 \qquad \text{SO} \qquad \omega_0 = \sqrt{\frac{g\rho_{H_2O}}{\rho_{Ducky}\,h}} = \sqrt{\frac{g\rho_{H_2O}\pi r^2}{m}} = \sqrt{\frac{g}{y_0}}$$

SOME OF THE WAYS TO EXPRESS $\omega_0$

(10) WE WANT REDUCE $A(\omega)$ AS MUCH AS POSSIBLE. SO WE WANT TO BE OFF RESONANCE. IF WE KNOW THE DRIVING $\omega$(S) WE CAN ~~TO~~ CHOOSE TO MAKE THE PENDULUM LENGTH SO THAT $(\omega^2-\omega_0^2)^2$ IS ~~SM~~ LARGE. INCREASING mass, $m$ IS GOOD. LARGE $b$ CAN HELP.