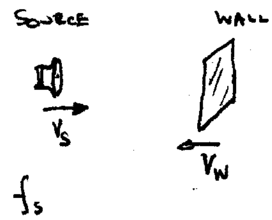
PHYS 195: WEEK 7 SOLUTIONS

(1.)



$$\begin{cases} V_s = 29.9 \text{ m/s} \\ V_w = 65.8 \text{ m/s} \end{cases}$$

IN THE WALL'S (MOVING!) FRAME

(a) $$f' = \left(\frac{c + V_W}{c - V_s}\right) f_s$$

$$= \frac{329 + 65.8}{329 - 29.9} \, 1200 \text{ Hz} \simeq 1584 \text{ Hz}$$

$$\simeq 1600 \text{ Hz.}$$

(b) IN THE WALL'S FRAME THE SPEED OF SOUND IS $c + V_W$

SO $$\lambda' = \frac{c + V_W}{f'} = \left(\frac{c + V_W}{c + V_W}\right)(c - V_s) f_s^{-1}$$

$$\left[= \lambda\left(1 - \frac{V_s}{c}\right)\right] = (1200)^{-1}(329 - 29.9)$$

$$\simeq 0.249 \text{ m} \simeq 0.25 \text{ m}$$

(c.) THE WAVE OF FREQUENCY f' REFLECTS OFF THE MOVING WALL. SO THE REFLECTED WAVE IS DOPPLER SHIFTED BY THE WALL'S MOTION AS WELL AS THE "SOURCE", NOW ACTING AS AN OBSERVER, HENCE

$$f'' = f'\left(\frac{c + V_s}{c - V_W}\right) = f_s\left(\frac{c + V_W}{c - V_s}\right)\left(\frac{c + V_s}{c - V_W}\right) \simeq (1584)\left(\frac{329 + 29.9}{329 - 65.8}\right) \text{ Hz}$$

$$\simeq 2159.9 \text{ Hz} \simeq 2.2 \times 10^3 \text{ Hz}$$

(d.) AS ABOVE $$\lambda'' = \frac{c + V_s}{f''} \simeq 0.166 \text{ m} \simeq 0.17 \text{ m}$$

(2.) (a) THE SYNODIC PERIOD BEATS!

(b) USING 3 BEAT PERIODS I FIND

$$T_{BEAT} \simeq 8.85 \text{ yrs} \simeq 3290 \text{ days}$$

LIKEWISE, THE HIGH FREQUENCY "SIGNAL" HAS A PERIOD OF $T\left[\simeq 1\,yr\right] \simeq 387$ days.

FROM $f = \frac{f_1+f_2}{2}$ AND $f_{BEAT} = f_1 - f_2$ WE HAVE

$$T_B = \frac{T_1 T_2}{T_1 - T_2} \quad \text{AND} \quad T = \frac{2T_1 T_2}{T_1 + T_2} \quad \text{SO THAT}$$

$$T_2 = \frac{2T\,T_B}{T + 2T_B} = \frac{(2)(387)(3290)}{387 + (2)(3290)} \simeq \underline{\underline{366 \text{ days}}} \simeq 1 \text{ YR !}$$

AND, FROM $\frac{2}{T} - \frac{1}{T_B}$, I FIND

$$T_1 = \frac{2T\,T_B}{2T_B - T} = \frac{(2)(387)(3290)}{(2)(3290) - 387} \simeq \underline{\underline{412 \text{ days}}}$$

YOUR NUMBERS MAY DIFFER FROM THESE. BTW THE BEAT PERIOD IS THE TIME BETWEEN ECLIPSES.

(3.) (a) THE WHALE ACTS AS A MOVING OBSERVER SO THE FREQUENCY IS SHIFTED IN THE WHALE' FRAME

$$f_W = f\left(\frac{c_s + V}{c_s}\right)$$

THIS SOUND IS REFLECTED OFF THE MOVING WHALE. WITH THIS MOVING "SOURCE" WE HAVE THE FREQUENCY AT THE RECEIVER GIVEN AS

$$f' = f\left(\frac{c_s + V}{c_s - V}\right)$$

AS EXPECTED.

(b) I'LL USE $(1+x)^n \simeq 1 + nx$ SO

$$\frac{f'}{f} = \frac{1 + \frac{V}{c_s}}{1 - \frac{V}{c_s}} = \left(1+\frac{V}{c_s}\right)\left(1-\frac{V}{c_s}\right)^{-1} \simeq \left(1+\frac{V}{c_s}\right)\left(1-\left(-\frac{V}{c_s}\right)\right)$$

$$\simeq 1 + 2\frac{V}{c_s} + \underbrace{\mathcal{O}\left(V^2/c_s^2\right)}$$

MEANING TERMS QUADRATIC ORDER OR HIGHER

(c) $\frac{f'}{f} \simeq 1 + 2\frac{V}{c_s} \simeq 1 + 2\left(\frac{1.00}{343}\right) \Rightarrow f' \simeq \underline{\underline{24.1\ kHz}}$,

USING $f = 24.06$ HZ.

(4.) THE TAPE EXPERIMENTS: I. WHEN THE TAPE IS PULLED UP IT (TYPICALLY) ATTRACTS EVERYING. I'LL CALL THE TAPE 'BOB'. THE TABLE IS FILLED WITH ATTRACTION (A) <u>EXCEPT</u> FOR BOB CLONES:

	BOB
COFFEE MUG	A
PEN	A
CHAIR	A
HRW	A
BOB II	R ← FOR REPEL

II THE NEW TABLE HAS TWO REPULSIONS, BUT IS MOSTLY FILLED WITH ATTRACTION DUE TO CHARGING BY INDUCTION

	B	T
B	R	A
T	A	R
COFFEE MUG	A	A
⋮	A	A
	⋮	⋮

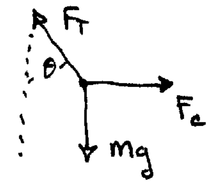
SO REPULSION IS THE KEY TO DIFFERENTIATE CHARGES. THE ONLY WAY TO COMPARE CHARGES IS TO COMPARE THEM WITH OTHER CHARGED OBJECTS, LOOKING FOR REPULSION (IF THEY REPEL THEN THEY ARE THE SAME CHARGE.) THERE IS NO WAY, EVEN IN PRINCIPLE, TO DETERMINE 'WHICH CHARGE IS WHICH' WITHOUT COMPARING THEM TO A CHARGE YOU (OR SOMEONE ELSE) HAVE NAMED e.g. "BOB" OR "POSITIVE" OR "NEGATIVE". THIS FACT IS A DEEP PRINCIPLE OF MODERN PHYSICS KNOWN AS LOCAL GAUGE INVARIANCE!

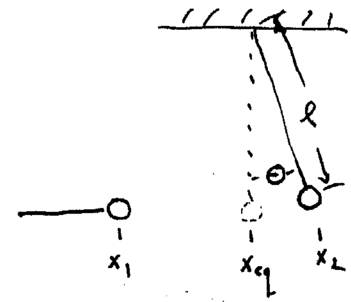
THE STRENGTH OF THE INTERACTION IS EASILY SEEN BY HOW MUCH THE TAPE BENDS AS THEY ARE BROUGHT TOGETHER. THE INTERACTION IS STRONGER WHEN OBJECTS ARE CLOSE TOGETHER AND FOR 'VIGOROUSLY PREPARED' TAPE STRIPS. WE COULD GET A QUANTITATIVE MEASURE OF THE STRENGTH OF INTERACTION BY MEASURING THE EXTENT OF THE DEFLECTION, AS WE DO IN THE NEXT PROBLEM...

(5.) FROM THE GEOMETRY

WE HAVE

(a)





EQUILIBRIUM MEANS $\Sigma \vec{F} = 0$ SO

$$\begin{cases} -F_T \sin\theta + F_e = 0 \quad \text{AND} \\ -mg + F_T \cos\theta = 0 \end{cases}$$

FOR SMALL ANGLES, $\sin\theta \simeq \theta$ AND $\cos\theta \simeq 1$ SO $|F_T| = mg$

AND WE ARE LEFT WITH

$$-mg\theta + F_e = 0 \quad \text{AND WITH} \quad F_e = \frac{K}{r^n}$$

$$\Rightarrow \quad \frac{K}{r^n} = mg\theta. \quad \text{AGAIN, FROM THE GEOMETRY}$$

$$r = x_2 - x_1 \quad \text{AND} \quad \theta \simeq \frac{x_2 - x_{eq}}{\ell}$$

IT IS USEFUL TO CREATE SPREADSHEET COLUMNS WITH THE r AND θ VALUES. YOU NEED TO ASSUME (OR MEASURE!) ℓ. IT IS ABOUT 40 cm. FINALLY IT IS GOOD TO TAKE THE LOG SO WE CAN ISOLATE "n". HENCE

$$\frac{1}{r^n} = \frac{mg}{K}\theta \quad \text{OR} \quad \theta = \frac{K}{mg} r^{-n}$$

AND $$\ln\theta = -n\ln(r) + \ln(K/mg)$$

(b) AND (c) SEE SPREADSHEET

Coulomb Data 2009

Positions (to center):
all measurements in cm

Pendulum ball "x2"	Ball on stick "x1"	r	ln r	theta	ln theta
15.4	3.1	12.3	-2.5096	0.004831	-5.332719
15.6	5	10.6	-2.36085	0.009662	-4.639572
16	8.4	7.6	-2.02815	0.019324	-3.946424
16.9	11.4	5.5	-1.70475	0.041063	-3.192653
17.5	13.2	4.3	-1.45862	0.055556	-2.890372
17.9	13.8	4.1	-1.41099	0.065217	-2.730029
18.1	14.5	3.6	-1.28093	0.070048	-2.65857
18.7	15	3.7	-1.30833	0.084541	-2.470518

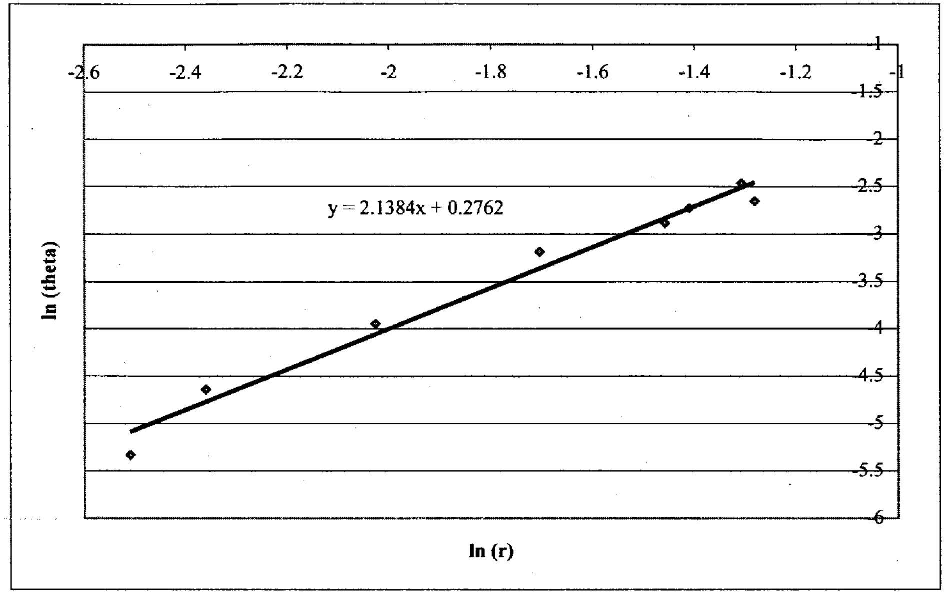
	slope	y-intercept
	2.13836	0.276155
uncertainty	0.12889	0.233971

Equilibrium of pendulum
15.2

Pendulum length
41.4

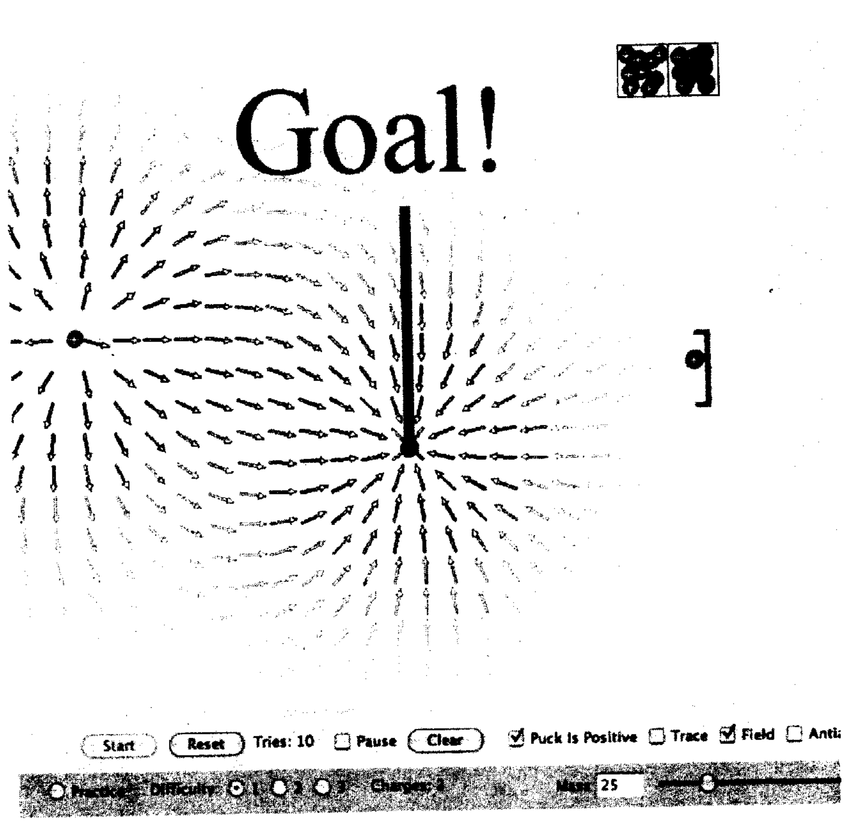
Uncertainty
0.1

So THE RESULT IS 2.1 ± 0.1 WHICH AGREES WITH $n=2$



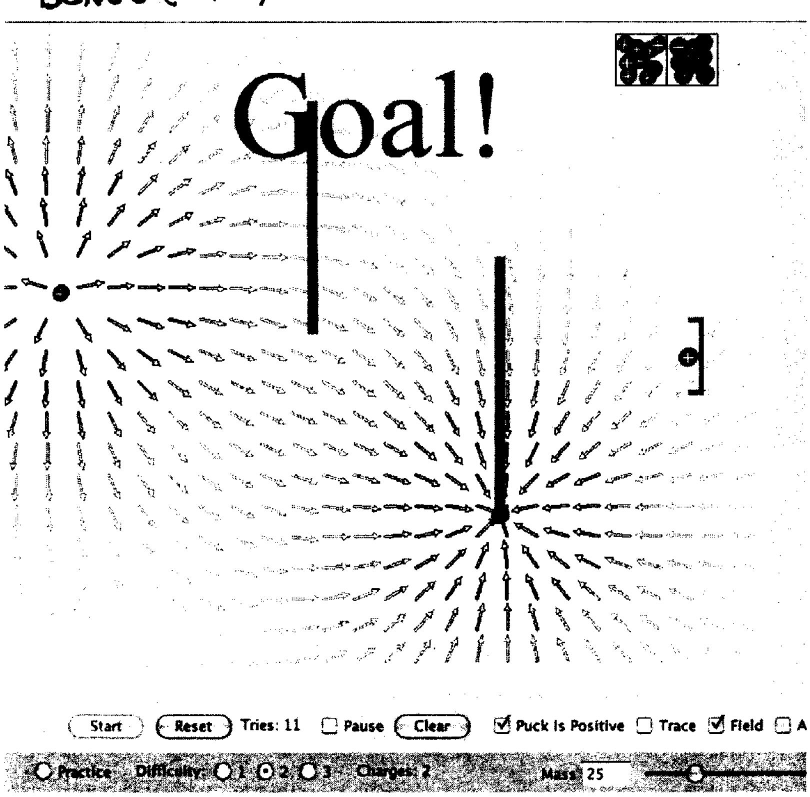
(7.) BOTH THESE CAN BE SOLVED WITH DIPOLE CONFIGURATIONS. SO THE (MOST ELEGANT) SOL'N IS WITH TWO CHARGES. THE + CHARGE ACTS AS A 'STICK' GIVING THE PUCK KINETIC ENERGY. THE – CHARGE CURVES THE TRAJECTORY TO END IN THE GOAL. THIS IS MUCH LIKE WHAT IS DONE

(a)



BONUS (1 PT.)

(b)

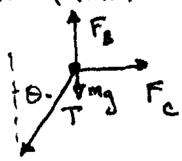


WITH SPACE CRAFT WHEN THEY UNDERGO 'FLY-BY'S' NEAR PLANETS.

[ONLY 2 CHARGE SOL'NS RECEIVE FULL CREDIT]

(8) BONUS (1 PT.) HERE ARE THE FREE BODY DIAGRAMS

BALLOON (RIGHT SIDE)

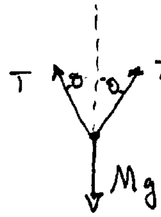


$\Rightarrow$ EQUILIBRIUM:

$$-T\cos\theta - mg + F_B = 0 \qquad (1)$$

$$-T\sin\theta + F_c = 0 \qquad (2)$$

WHALE

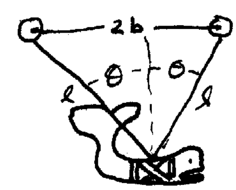


EQUILIBRIUM

$$\Rightarrow \quad -Mg + 2T\cos\theta = 0 \qquad (3)$$

FROM EQU'N (3) WE HAVE $T\cos\theta = \frac{Mg}{2}$. USING COULOMB'S FORCE AND THIS RESULT EQU'N (2) GIVES

$$-\frac{Mg}{2}\tan\theta + \frac{Q^2}{4\pi\epsilon_0 (2b)^2} = 0$$



$$\Rightarrow \quad Q^2 = 2\pi\epsilon_0 Mg \tan\theta\, (2b)^2$$

NOW SINCE $\cos\theta = \frac{\sqrt{l^2-b^2}}{l}$, $\sin\theta = \frac{b}{l}$ THEN

$$\tan\theta = \frac{b}{\sqrt{l^2-b^2}}$$

SO

$$Q^2 = \frac{8\pi\epsilon_0 Mg\, b^3}{\sqrt{l^2-b^2}} \quad \Rightarrow \quad Q = \sqrt{\frac{8\pi\epsilon_0 Mg\, b^3}{\sqrt{l^2+b^2}}}$$