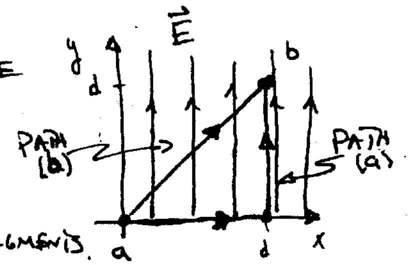
PHYS 195 : WEEK 8 SOLUTIONS

(1.) HERE'S THE PICTURE



(a) IN THIS CASE THE PATH HAS TWO SEGMENTS.

ON THE FIRST, $d\vec{l} = \hat{i}\,dx$ SO $\vec{E}\cdot d\vec{l} = E_0 dx\, \hat{i}\cdot\hat{j} = 0$

ON THE SECOND, $d\vec{l} = \hat{j}\,dy$ SO $\vec{E}\cdot d\vec{l} = \hat{j}\,dy \cdot E_0\hat{j} = E_0 dy$

$$\Rightarrow V = -\int_a^b \vec{E}\cdot d\vec{l} = -\left[\int_{x=0}^{x=d} \cancel{\vec{E}\cdot d\vec{l}}^{\,0} + \int_{y=0}^{y=d} \vec{E}\cdot d\vec{l}\,\right] = -\int_0^d E_0\,dy = \underline{\underline{-E_0 d}}$$

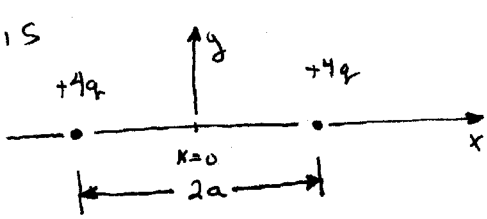
(b.) NOW $d\vec{l} = \dfrac{\hat{i}+\hat{j}}{\sqrt{2}}\,dl$ ALONG THE DIAGONAL PATH. ~~SO~~ THE VECTOR $\dfrac{\hat{i}+\hat{j}}{\sqrt{2}}$ IS THE UNIT VECTOR ALONG THE PATH. SO

$$\vec{E}\cdot d\vec{l} = \frac{E_0}{\sqrt{2}}\,dl \;.\quad \text{HENCE} \quad V = -\int_{(0,0)}^{(d,d)} \vec{E}\cdot dl = -\int_0^{\sqrt{2}d} \frac{E_0}{\sqrt{2}}\,dl$$

$$= -\frac{E_0}{\sqrt{2}}\sqrt{2}d = -E_0 d.$$

(c.) YES, THESE RESULTS MUST BE EQUAL FOR CONSERVATIVE FIELDS. (V (AND, LIKEWISE, U) IS ONLY DEFINE FOR CONSERVATIVE FORCES AND FIELDS.)

(2.) (a.) LET'S PLACE THE 2 +4q CHARGES ON A LINE – I'LL CALL IT THE X-AXIS – ~~AND~~ AT A DISTANCE 2a APART, LIKE THIS



I'LL FIND LOCATIONS WHERE $\vec{E}=0$, WHEN THE $-q$ CHARGE WILL BE IN EQUILIBRIUM. ALONG THE X-AXIS,

$$\vec{E} = \frac{4q}{4\pi\epsilon_0 (a+x)^2}(\hat{\imath}) + \frac{4q}{4\pi\epsilon_0 (x-a)^2}(-\hat{\imath})$$

$$= \left(\frac{q\hat{\imath}}{\pi\epsilon_0}\right)\left(\frac{\cancel{x^2} - 2ax + \cancel{a^2} - \cancel{a^2} - 2ax - \cancel{x^2}}{(-a^2+x^2)^2}\right)$$

$$= \frac{-2axq\hat{\imath}}{\pi\epsilon_0 (x^2-a^2)^2}$$

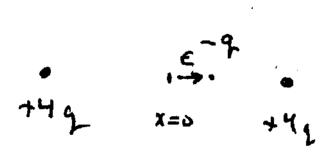
WELL THIS VANISHES WHEN $x=0$! TO CHECK THAT THE $+4q$ CHARGES ARE <u>ALSO</u> IN EQUILIBRIUM, CONSIDER THE ONE ON THE RIGHT. THE $\vec{E}$-FIELD IS

$$\vec{E}_{+4q}(a) = \frac{4q}{4\pi\epsilon_0 (a+a)^2}\hat{\imath} - \frac{q}{4\pi\epsilon_0 a^2}\hat{\imath} = \left(\frac{q\hat{\imath}}{4\pi\epsilon_0}\right)\left(\frac{4}{(2a)^2} - \frac{1}{a^2}\right) = 0 \checkmark$$

SINCE $E(a)=0$ THEN THIS $+4q$ CHARGE IS IN EQUILIBRIUM. BY SYMMETRY THIS ALSO HOLDS FOR THE $+4q$ AT $x=-a$. ALL CHARGES ARE IN EQUILIBRIUM.

(b.) TO SEE WHETHER THE $-q$ CHARGE IS STABLE LET'S DISPLACE IT SLIGHTLY TO THE RIGHT BY AMOUNT ϵ. IF THE RESULTING FORCE IS RESTORING (OF THE FORM $F=-k\epsilon$) THEN THE EQUILIBRIUM IS STABLE. THE FORCE IS THEN

$$\vec{F}_{NET}(\epsilon) = \left(\frac{1}{4\pi\epsilon_0}\right)\left[\frac{4q^2}{(a-\epsilon)^2}\hat{\imath} - \frac{4q^2}{(a+\epsilon)^2}\hat{\imath}\right]$$



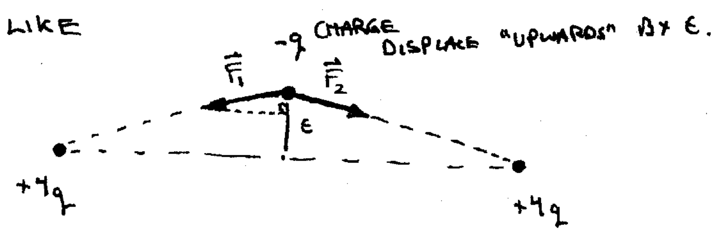
$$\vec{F}_{NET}(\epsilon) = \left(\frac{q^2\hat{i}}{\pi\epsilon_0}\right)\left[\frac{1}{(a-\epsilon)^2} - \frac{1}{(a+\epsilon)^2}\right]$$

$$= \left(\frac{q^2\hat{i}}{\pi\epsilon_0 a^2}\right)\left[\frac{1}{\left(1-\frac{\epsilon}{a}\right)^2} - \frac{1}{\left(1+\frac{\epsilon}{a}\right)^2}\right].$$ NOW TO LEADING ORDER

$$\simeq \left(\frac{q^2\hat{i}}{\pi\epsilon_0 a^2}\right)\left(1 + \frac{2\epsilon}{a} - 1 + \frac{2\epsilon}{a}\right) + O\left(\frac{\epsilon^2}{a^2}\right)$$

$$= \frac{q^2\hat{i}}{\pi\epsilon_0 a^2}\left(\frac{4\epsilon}{a}\right) > 0.$$ SO IF THE CHARGE IS

DISPLACED TO THE RIGHT THE FORCE IS TO THE RIGHT; THE CHARGE ACCELERATES AWAY FROM $x=0$! THIS IS AN UNSTABLE EQUILIBRIUM. TO BE COMPLETE WE SHOULD LOOK AT THE y-DIRECTION AS WELL. DISPLACING THE CHARGE WOULD LOOK LIKE



FROM THE GEOMETRY,

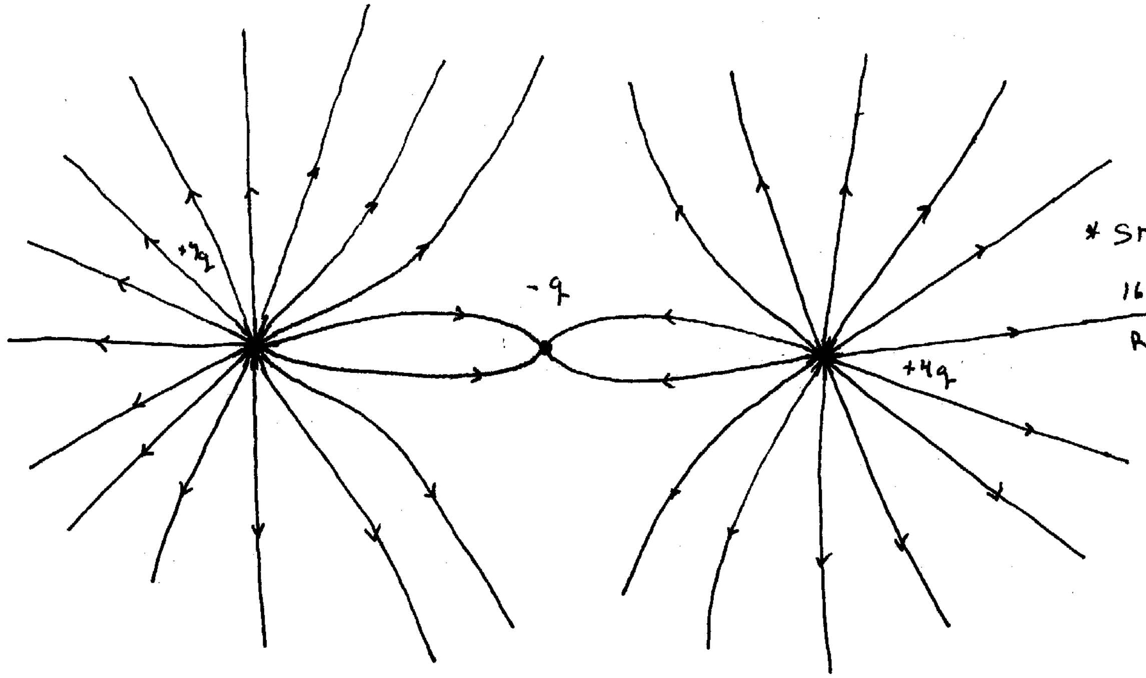
$$\vec{F}_{NET} = -2|F|\sin\theta\,\hat{j} = \frac{-2\cdot 4q^2}{4\pi\epsilon_0(a^2+\epsilon^2)}\cdot\frac{\epsilon}{\sqrt{a^2+\epsilon^2}}\,\hat{j} \cong \frac{-2q^2}{\pi\epsilon_0 a^3}\,\epsilon\,\hat{j}.$$

THIS IS OF THE FORM "$\vec{F} = -k_{eff}\,x$" WITH $k_{eff} = \frac{2q^2}{\pi\epsilon_0 a^3}$

SO THE CHARGE IS STABLE IN THE y-DIRECTION.

A SIMILAR ANALYSIS CAN BE DONE FOR THE $+4q$ CHARGES.

(3.)



* SHOULD BE 16 LINES RATHER THAN 14!

(4.) CASES (a) AND (c) ARE SIMILAR IN THAT THE FORCE ($q\vec{E}$) IS CONSTANT FROM A TO B. IN CASE (c.) THE E-FIELD IS WEAKER SINCE THE FIELD LINES ARE FURTHER APART. IN CASE (b) THE FIELD CHANGES, GOING FROM APPROXIMATELY THE STRENGTH IN (a) TO THE STRENGTH IN (c) AS THE PROTON TRAVELS FROM A TO B. HENCE THE MOMENTUM AT B WOULD BE GREATEST FOR (a.) THEN (b) THEN, WITH THE LEAST, (c.)

(5.) THIS IS A PROJECTILE MOTION PROBLEM WITH A UNIFORM FORCE $q\vec{E}$ IN THE +y-DIRECTION. SO, $\vec{a} = \frac{q\vec{E}}{m} = \frac{-(1.6\times10^{-19}C)(5\ N/C)\hat{j}}{9.1\times10^{-31}kg}$ AND

$\vec{V}(t) = V_y(t)\hat{j} + V_x(t)\hat{i}$ WITH

$$\begin{cases} V_x(t) = V_0\cos\theta_0 = \left(2\times10^6\ \frac{m}{s}\right)\cos(40^\circ) \\ V_y(t) = V_0\sin\theta_0 + at = V_0\sin\theta_0 + \frac{qE}{m}t = \left(2\times10^6\ \frac{m}{s}\right)\sin(40^\circ) + at. \end{cases}$$

THE e- HITS THE SCREEN WHEN

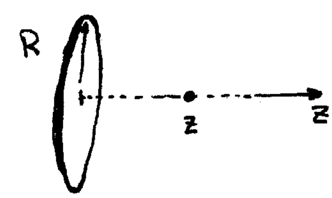
$$3m = V_x t_* = V_0\cos\theta_0\, t_* \Rightarrow t_* = \frac{3m}{V_0\cos\theta_0}$$
$$\simeq 1.96\times10^{-6}s$$

OR

$$\vec{V}(t_*) = V_0\cos\theta_0\,\hat{i} + \left(V_0\sin\theta_0 + \frac{qE}{m}\frac{3}{V_0\cos\theta_0}\right)\hat{j}$$

$$\simeq \left(1.53\times10^6\,\hat{i} - 4.38\times10^5\,\hat{j}\right)\ \mathrm{m/s} \qquad [q_{e^-} < 0!]$$

(6.) (a) $\vec{E}(z) = \dfrac{-Qz\,\hat{k}}{4\pi\epsilon_0(z^2+R^2)^{3/2}}$



(b) SO $F(z) = qE = \dfrac{-qQz}{4\pi\epsilon_0(z^2+R^2)^{3/2}}\hat{k}$

IS CLEARLY NOT IN THE FORM "$-kx$". BUT IF z IS SMALL

$(z \ll R)$ THEN) $(z^2+R^2)^{-3/2} = R^{-3}\left(\frac{z^2}{R^2}+1\right)^{-3/2} \simeq R^{-3}\left(1-\frac{3z^2}{2R^2}\right)$

SO UP TO ORDER $\frac{z^2}{R^2}$ WE HAVE

$$\vec{F}(z) = -\frac{qQz}{4\pi\epsilon_0 R^3}\hat{k}$$ — NEAT! NOW THIS IS A

HOOKE'S LAW-TYPE FORCE, FROM $F=ma$

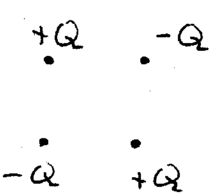
$$m\frac{d^2z}{dt^2} = \frac{-qQz}{4\pi\epsilon_0 R^3} \quad \text{OR} \quad \boxed{\frac{d^2x}{dt^2} + \frac{qQ}{4\pi\epsilon_0 a^3 m}z = 0} \quad \text{SHM!}$$

$$\Rightarrow \omega^2 = \frac{qQ}{4\pi\epsilon_0 a^3 m} \quad \text{AND SO} \quad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{4\pi\epsilon_0 m a^3}{qQ}}$$

$$= 4\sqrt{\frac{\pi^3 a^3\epsilon_0 m}{qQ}}$$

(7.) SEE NEXT PAGE FOR THE PICTURE

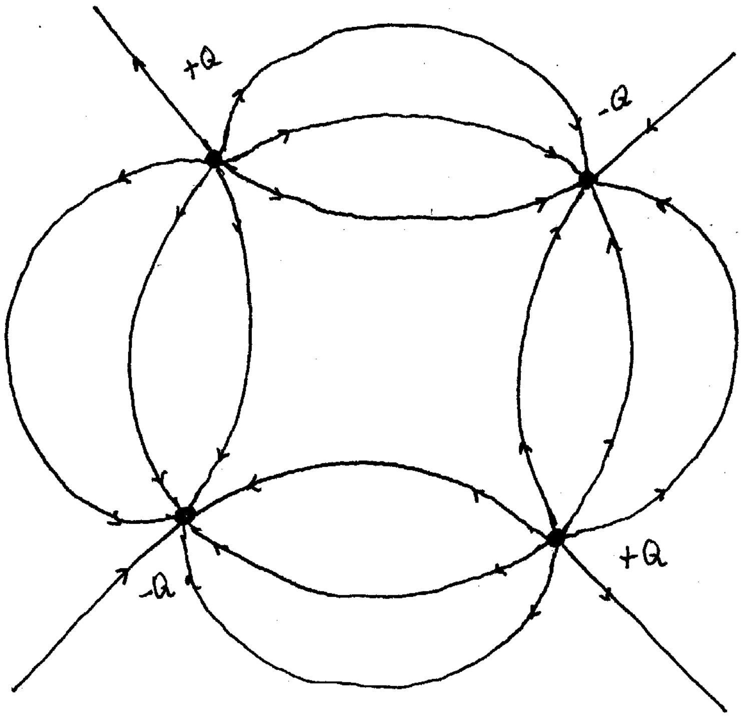
(b) THE QUADRUPOLE HAS CONFIGURATION



THAT IS, CHARGES ARE IN A SQUARE

AND OPPOSITE, DIAG. CHARGES ARE EQUAL.

(a)



(C.) THE FIELD LINES CURVE IN 3D SPACE SO NOT ONLY DO THEY BEND AS I'VE SKETCHED BUT FIELD LINES THAT COME OUT OF THE PAGE BEND DOWN AS WELL, GIVING AN ACCURATE VISUAL DESCRIPTION OF THE ACTUAL ELECTRIC FIELD. A 2D PICTURE IS JUST A PROJECTION OF THIS MORE COMPLICATED STRUCTURE.

(8.) (a.) THE ELECTRIC POTENTIAL IS, TAKING $V(\infty)=0$,

$$V(\vec{r}) = \frac{Q_1}{4\pi\epsilon_0|\vec{r}_1-\vec{r}|} + \frac{Q_2}{4\pi\epsilon_0|\vec{r}_2-\vec{r}|} + \frac{Q_3}{4\pi\epsilon_0|\vec{r}_3-\vec{r}|}$$

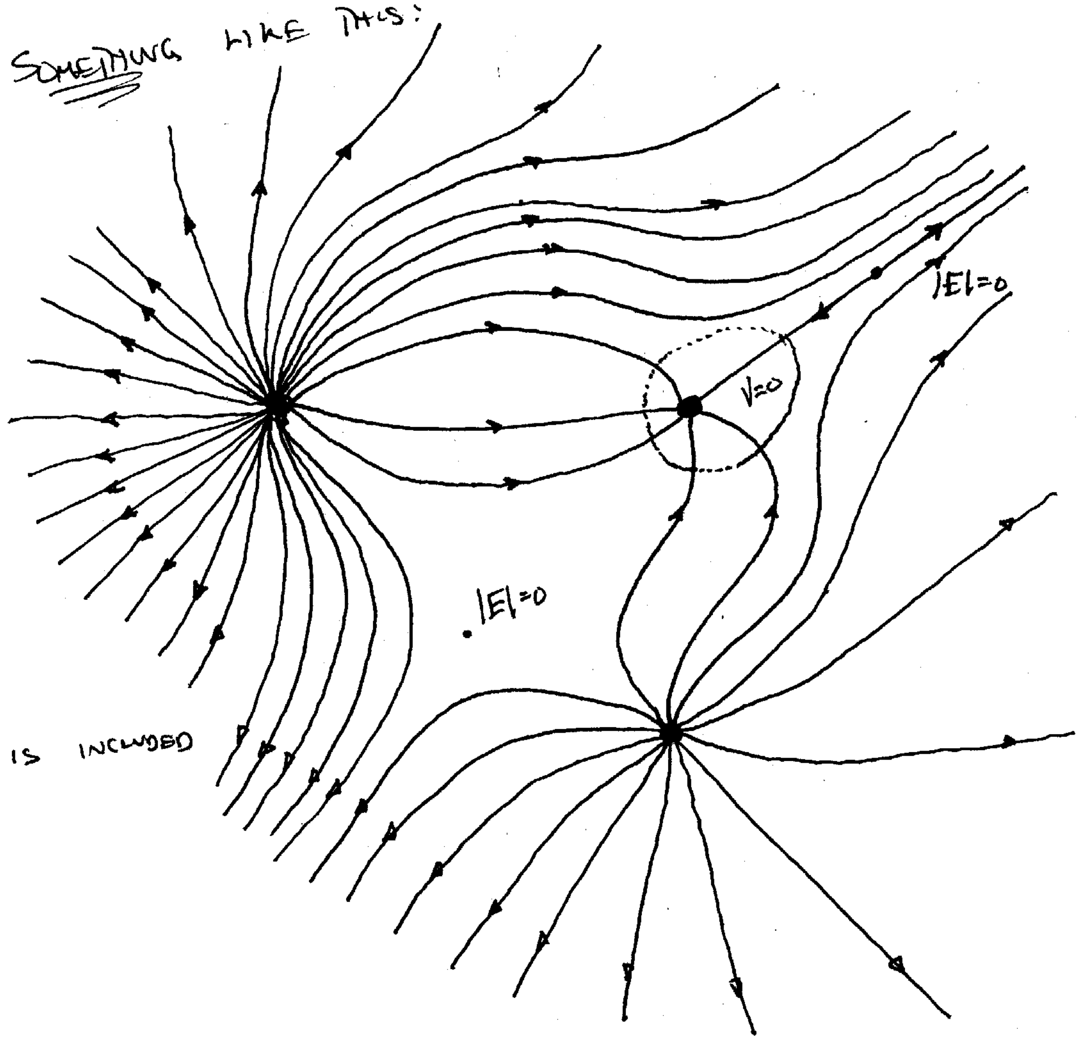
FOR P_2, ALL THE DISTANCES ARE $\frac{a}{\sqrt{2}}$. SO AT P_2

$$V = \frac{Q_1+Q_2+Q_3}{4\pi\epsilon_0\left(\frac{a}{\sqrt{2}}\right)} \simeq 7.6 \times 10^4 \text{ V}$$

FOR P_3, $|\vec{r}_1 - \vec{r}_{P_3}| = 3$, $|\vec{r}_2 - \vec{r}_{P_3}| = 2$ AND $|\vec{r}_3 - \vec{r}_{P_3}| = \sqrt{5}$ SO

AT P_3, $V = 1.9 \times 10^4 V$

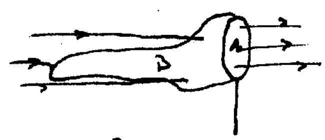
(b.) SOMETHING LIKE THIS:



(c) IS INCLUDED

(9.) ~~Be~~ THERE'S NO CHARGE INSIDE SO $\Phi_{\text{WHOLE SURFACE}} = 0$

THE FLUX THROUGH THE NET MUST BE − (THE FLUX THROUGH THE RIM SURFACE



HERE, $\Phi_A = EA$

$= \pi r^2 E = (\pi)(11\,\text{cm})^2 (3 \times 10^{-3}\,\text{N/C})$

SINCE $\Phi_{\text{WHOLE SURFACE}} = \Phi_A + \Phi_B = 0$ THEN $\Phi_B = -\Phi_A = -\pi r^2 E \simeq 1.1 \times 10^{-4}\,\text{Nm}^2\text{C}^{-1}$

(10.) (a) $V_B - V_A = \frac{W}{q} = \frac{W}{-e} \simeq \frac{-3.99 \times 10^{-19}\,J}{-1.6 \times 10^{-19}\,C} \simeq 2.46\,V$

WORK DONE ON e^-

(b.) SAME AS (a)

(c.) $V_C - V_B = 0$ SINCE THEY ON AN EQUIPOTENTIAL