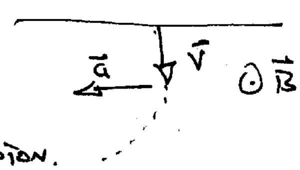
PHYS 195 : WEEK 11 SOLUTIONS

(1.) IN SERIES, RESISTANCES ADD SO $R_{eq} = 4 \cdot 90\,\Omega = 360\,\Omega \simeq 400\,\Omega$

IN PARALLEL, RECIPROCALS ADD SO $\dfrac{1}{R_{eq}} = \dfrac{4}{90} \Rightarrow R_{eq} \simeq 22.5 \simeq 20\,\Omega$

FOR n RESISTORS OF RESISTANCE R, $R_{eq} = nR$ IN SERIES AND $R_{eq} = \dfrac{R}{n}$ IN PARALLEL. IF A BULB BURNS OUT IN A SERIES CIRCUIT THE CURRENT STOPS AND ALL THE BULBS GO OUT. IN A PARALLEL CIRCUIT IF ONE BULB GOES OUT, CURRENT STILL FLOWS TO THE OTHER BULBS AND THEY WILL STAY LIT. ~~(THEY'LL DIM'L LSONG~~ SO TO MAKE A MORE ROBUST STRING OF LIGHTS, USE A PARALLEL CIRCUIT CONSTRUCTION.

(2.) FROM THE DIAGRAM



SO $\vec{V} \times \vec{B}$ POINTS IN THE DIRECTION OF $\vec{a}$ SO $q > 0$.

HENCE THE PARTICLE IS A PROTON.

FROM $F = ma$,

$$q\vec{V} \times \vec{B} = qVB = \frac{m_p V^2}{r} \Rightarrow eB = \frac{m_p V}{r} \quad \text{OR} \quad B = \frac{m_p V}{er}$$

NOW $V = \dfrac{2\pi r}{T}$ ("ONE CIRCUMFERENCE IN ON PERIOD") SO

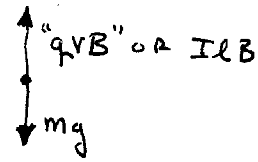
(a.) $$B = \frac{m_p 2\pi \cancel{r}}{e \cancel{r} T} = \frac{m_p \pi}{e\,T/2} = \frac{(1.673\times10^{-27})(\pi)}{(1.602\times10^{-19})(130\times10^{-9}\,s)}$$

$$\simeq 0.25\ T$$

(b.) IF THE KINETIC ENERGY DOUBLES THEN THE SPEED INCREASES BY $\sqrt{2}$. SINCE $r = \dfrac{m_p V}{eB}$, THE RADIUS ALSO INCREASES BY $\sqrt{2}$. THUS, THE PERIOD REMAINS THE SAME

AND THE TIME, $T/2$, DOESN'T CHANGE.

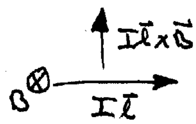
(3.) MAGNETIC LEVITATION! HERE'S THE FBD



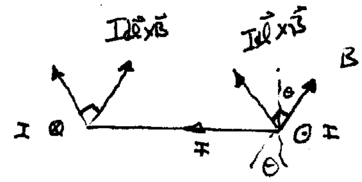
(a) EQUILIBRIUM MEANS $mg = IlB$

$$\Rightarrow \quad I = \frac{mg}{lB} = \frac{(13\,g)(9.8\,m/s^2)}{(.62\,m)(0.44\,T)} \simeq 0.467\,A$$

(b) FOR LIFT I MUST FLOW FROM LEFT TO RIGHT



(4.) IN THE PLANE OF THE LOOP



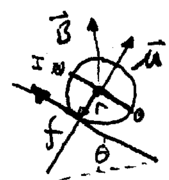
SO $F = I\vec{l}\times\vec{B}$ POINTS AS SHOWN. FROM THE GEOMETRY IT IS CLEAR THAT THERE IS A NET UPWARDS FORCE OF MAGNITUDE

$$F = I\int d\vec{l}\times B\cos(90-\theta) = 2\pi r\, I B\cos(90-\theta) = 2\pi r I B\sin\theta$$

$$\simeq (2\pi)(1.8\,cm)(4.6\,mA)(3.4\,mT)\sin(20^\circ) \simeq 6.0\times10^{-7}\,N$$

THE INTEGRAL IS SIMPLE BECAUSE I AND $\vec{B}$ ARE CONSTANT AROUND THE LOOP.

(5.) HMM, SO WE WANT ZERO NET TORQUE. THE TORQUE AROUND THE AXIS OF THE CYLINDER ARISES FROM THE MAGNETIC MOMENT AND THE FRICTION BETWEEN THE INCLINE PLANE AND THE CYLINDER



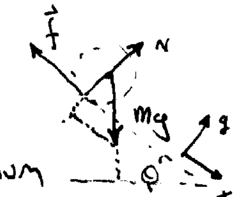
DIRECTIONS

$\vec{\mu}\times\vec{B}$ ⊙ ← POSITIVE

$\vec{r}\times\vec{f}$ ⊗

SO $\vec{\tau}_B + \vec{\tau}_f = 0$ WITH $\vec{\tau}_B = \vec{\mu}\times\vec{B}$

AND $\vec{\tau} = \vec{r}\times\vec{f}$. TO FIND $\vec{f}$ WE NEED TO STUDY THE FORCES SO THE FBD LOOKS LIKE



IN THE x DIRECTION, $+mg\sin\theta - f = 0$ FOR EQUILIBRIUM

$\Rightarrow$ $f = mg\sin\theta$. HENCE, FROM THE TORQUE

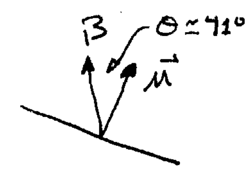
$$\mu B\sin\theta - rmg\sin\theta = 0$$

OR

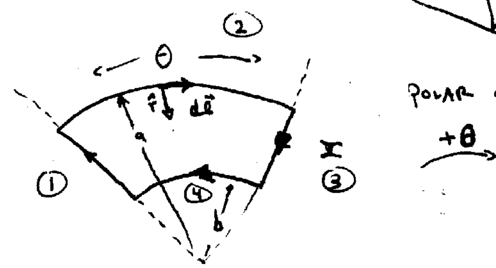
$$I N \overbrace{L(2r)}^{\text{AREA}} B \sin\theta - mgr\sin\theta = 0$$

$$\Rightarrow I = \frac{mg\cancel{r\sin\theta}}{2NLB\cancel{r\sin\theta}} = \frac{mg}{2NLB} = \frac{(0.250\,kg)(9.8\,m/s^2)}{(2)(10)(0.1\,m)(.5\,T)} \simeq \underline{2.45\,A}$$

(6.) (a) $\mu = IA = I\pi r^2 \simeq 0.184\ A\,m^2$



(b) $|\vec{\tau}| = |\vec{\mu}\times\vec{B}| = \mu B\sin\theta = I\pi r^2 B\sin\theta \simeq 1.45\ Nm$



(7.) FOR THE FUNNY "LOOP" BIOT-SAVART GIVES

$$\vec{B} = \int \frac{\mu_0}{4\pi}\frac{I\,d\vec{l}\times\hat{r}}{r^2}$$

FOR SIDE ① $d\vec{l} = dr\,\hat{r}$ so $d\vec{l}\times\hat{r} = 0$ – NO CONTRIBUTION FROM THIS SIDE. LIKEWISE FOR SIDE ③ $d\vec{l} = -dr\,\hat{r}$ so $d\vec{l}\times\hat{r} = 0$ FOR THIS SIDE AS WELL.

FOR SIDE ② $d\vec{l} = a\,d\theta\,\hat{\theta}$ so $d\vec{l}\times\hat{r} = a\,d\theta$ (DOWN) AND

$$\vec{B}_{②} = \int_0^{\theta} \frac{\mu_0}{4\pi}\frac{I\,a\,d\theta}{a^2} = \frac{\mu_0 I}{4\pi a}\theta \text{ DOWN}$$

SIMILARLY FOR SIDE 4, $d\vec{l} = -b\,d\theta\,\hat{\theta}$ so $d\vec{l}\times\hat{r} = b\,d\theta$ (UP)

$$\vec{B}_{④} = \int_0^{\theta} \frac{\mu_0}{4\pi}\frac{I\,b\,d\theta}{b^2} = \frac{\mu_0 I\theta}{4\pi b} \text{ UP}$$

FOR ALL SIDES,

$$\vec{B} = \frac{\mu_0 I\theta}{4\pi}\left(\frac{1}{b} - \frac{1}{a}\right) \underline{\underline{UP}} \simeq \left(\frac{4\pi\times10^{-7}}{4\pi}\right)(0.411\,A)\left(74^\circ\frac{\pi}{180^\circ}\right)\left(\frac{1}{0.107\,m} - \frac{1}{0.135\,m}\right)$$

$$= \underline{1.02\times10^{-7}\,T}$$

(8.) SINCE

$$B(z) = \frac{\mu_0 n I}{2}\left[\frac{z}{(z^2+R^2)^{1/2}} - \frac{z+L}{((z+L)^2+R^2)^{1/2}}\right]$$

$$= \frac{\mu_0 n I}{2}\left[\frac{\cancel{z}}{\cancel{z}\left(1+\frac{R^2}{z^2}\right)^{1/2}} - \frac{\cancel{z+L}}{\cancel{(z+L)}\left(1+\frac{R^2}{(z+L)^2}\right)^{1/2}}\right]$$

USING $(1+x)^n \simeq 1+nx$ WITH $n=-\frac{1}{2}$

$$\simeq \frac{\mu_0 n I}{2}\left[\cancel{1} - \frac{R^2}{2z^2} - \cancel{1} + \frac{R^2}{2(z+L)^2}\right]$$

$$= \left(\frac{\mu_0 n I R^2}{4}\right)\left[\frac{1}{(z+L)^2} - \frac{1}{z^2}\right]$$

THIS IS OF THE SAME FORM AS THE ELECTRIC FIELD OF TWO, ~~OPPOSITELY~~ OPPOSITE CHARGES – A DIPOLE! FOR $z \gg L$ AS WELL THE FIRST TERM IS $\frac{1}{z^2}\frac{1}{\left(1+\frac{L}{z}\right)^2} \simeq \frac{1}{z^2}\left(1-\frac{2L}{z}\right)$

So

$$B \simeq \left(\frac{\mu_0 n I R^2}{4}\right)\left[\frac{1}{z^2}\left(\cancel{1}-\frac{2L}{z}\right) - \cancel{\frac{1}{z^2}}\right] = -\frac{\mu_0 n I R^2 L}{z^3}$$

WHICH IS GOOD SINCE THE FIELD OF A DIPOLE FALLS OFF AS z^{-3} ~~ON THE~~ ALONG THE DIRECTION OF THE DIPOLE MOMENT.