Solutions:

(1) (a) By definition $T' = KT$ (using the notation we had in class). So since the frequency is inversely proportional to the period we have that the observed frequency $f_o = 1/T_o = 1/T'$ must be related to the emitted frequency $f_e = 1/T_e = 1/T$ via,

$$f_o = \frac{1}{K} f_e.$$

(b) Similarly since the wavelength is inversely related to the frequency $c = f\lambda$ we see that the redshift $z = \frac{\lambda_o - \lambda_e}{\lambda_e} = \frac{cT_o - cT_e}{cT_e} = K - 1 = \sqrt{1 + \frac{v}{c}} - 1.$$

There are other ways to express this such as $\gamma(1 + \frac{v}{c}).$

(c) The redshift is $z = \frac{\lambda_o - \lambda_e}{\lambda_e} \approx -0.18 < 0$ so the light is blueshifted.

After all you are approaching the light. Since $z = K - 1$ and

$$\frac{v}{c} = \frac{K^2 - 1}{K^2 + 1}$$

then $K \approx 0.82$ and $v \approx -0.2c \approx -6 \times 10^7 \text{ m/s}$. The velocity is negative indicating approach. This speed is approximately $1.3 \times 10^8 \text{ mph}$ - zippy! At $10^{-3}$ dollars per mph over 45 mph, this gives a fine of $130,000$ (assuming the judge takes into account sig figs).

(2) By conservation of energy for the process $p + D \rightarrow ^3\text{He} + \gamma$ the energy of the photon is

$$E_\gamma \approx (1.6724 + 3.3432 - 5.0058) \times 10^{-27}c^2 \approx 8.82 \times 10^{-13} \text{ J} \approx 55 \text{ MeV},$$

which lies in the gamma ray part of the spectrum. BTW the wavelength is about $2.26 \times 10^{-13} \text{ m}$ and the frequency is about $1.3 \times 10^{21} \text{ Hz}$.

(3) If the earth is bathed with $1350 \text{ W/m}^2$ then so is every other portion of a sphere with the (average) Earth’s orbital radius. This means that the sun produces

$$P = 4\pi r^2 I \approx 3.82 \times 10^{26} \text{ W}$$

of power. That’s one bright bulb. (I used a radius of $1.5 \times 10^8 \text{ km}$.) Using $E = mc^2$ to convert to mass gives about $4.2 \times 10^9 \text{ kg}$ per second, or $4.2 \times 10^6 \text{ metric tons per second}$.

(4) Optional Bonus - please submit to me for correcting.

(5) A relativistic $e^-$ in a parallel plate capacitor

(a) To find all these quantities we first need to find the electron’s energy. Since it has accelerated through a 250 keV potential then

$$E = mc^2 + e\phi = 500 + 250 = 750 \text{ keV}$$

so the total energy $E = \gamma mc^2 \implies \gamma = \frac{E}{mc^2} = \frac{3}{2}$

from which most of the rest follows.

$$\beta = \frac{v}{c} = \sqrt{1 - 1/\gamma^2} = \sqrt{5}/3 = 0.745.$$
\[ p_x = \gamma mv = \sqrt{5/2}mc \simeq 1.12mc. \]

The time between plates is \( t = 0.04/v \simeq 1.79 \times 10^{-10} \text{ s}. \)

The transverse force, which is constant, is equal to the rate of change of the transverse momentum

\[ p_y = \frac{eVt}{s} \simeq 0.08mc \]

where \( s \) is the separation between plates. The transverse velocity when the electron exists is given from the momentum

\[ p_y = \gamma mv_y \implies v_y \simeq 1.6 \times 10^7 \text{ m/s} \]

about an order of magnitude below \( v_x \simeq 2 \times 10^8 \text{ m/s} \) so that I will assume non-relativistic mechanics in this part. Due to the constant acceleration the average transverse velocity is one half of this or \( \bar{v}_y \simeq 8 \times 10^6 \text{ m/s}. \) The distance travelled is thus \( \Delta y = \bar{v}_y t \simeq 1.4 \text{ mm}. \)

The angle can be found from the momenta

\[ \theta = \frac{p_y}{p_x} \simeq 0.072 \text{ rad} \simeq 4.1^\circ. \]

(b) In the rest frame of the electron the speed is the same as above but the plates are contracted to \((0.04)/\gamma = 0.0267 \text{ m}. \) The electron enjoys the field \( E' = \gamma E \) for \( t' = 0.0267/0.745c \simeq 1.2 \times 10^{-10} \text{ s} \) and is accelerated upward. The upward momentum is \( eE't' = eEt = 0.08mc, \)

which is the same as the lab frame as it must be. The rest of the transverse results are the same due to the canceling of Lorentz contraction and time dilation.

(6) Done in text

(7) You can use \( dp = Fdt \) and a bit of geometry to find the result but I use the following method. From the Lorentz force we have

\[ \frac{d(\gamma mv)}{dt} = q\mathbf{v} \times \mathbf{B} \implies \frac{d\mathbf{v}}{dt} = \frac{q}{\gamma m} \mathbf{v} \times \mathbf{B} \]

since \( \gamma \) does change due to the transverse nature of the force. Let’s assume the magnetic field is uniform and that it points in the \( z \) direction then the \( x \) and \( y \) components are

\[ \frac{dv_x}{dt} = \frac{qB}{\gamma m} v_y \text{ and } \frac{dv_y}{dt} = -\frac{qB}{\gamma m} v_x. \]

Taking the derivative of the first and plugging in the second gives the familiar SHO equation

\[ \frac{d^2v_x}{dt^2} = -\left( \frac{qB}{\gamma m} \right)^2 v_x \]

with natural angular frequency

\[ \omega_0 = \frac{qB}{\gamma m} \text{ and period } T = \frac{2\pi \gamma m}{qB}. \]

The \( y \) equation yields essentially the same result. From here we can integrate up to find position, if we so desire.
(8) A 45 degree rotation requires equal components in the two perpendicular directions. So the wire would produce a $2 \times 10^{-5}$ T field at about 2 cm. Thus,

$$B = \frac{\mu_0 I}{2\pi r} \implies I = \frac{2\pi r B}{\mu_0} = 2A.$$  

(9) Due to the $F = I\ell \times B$ force, the force per unit length is $IB$ or

$$IB = \frac{\mu_0 I^2}{2\pi r} \simeq 1.6 \times 10^{-3} \text{ N/m}.$$  

It is repulsive.

(10) The 195 derivation is fine but I will present a more general result here. The loop is in the $xy$ plane. The $z$ component of the B-field is not going to produce torque around the $x$ or $y$ axes. To start let’s look at the $z$ component of the force on a wee length $d\ell$ of the loop, as drawn in figure 6.41

$$dF_z = I(d\ell \times B)_z = Id\ell B_y \sin \theta = Idx B_y$$

where the last equality is due to the geometry; the angle $\theta$ is between the $y$ direction and $d\ell$ and $d\ell \sin \theta$ is just $dx$. The torque on this bit is $d\tau_x = yIdxB_y$. Integrating up we have

$$\tau_x = \int d\tau_x = IB_y \int ydx = IaB_y,$$

where $a = \int ydx$ is the area of the loop. Now, you might be wondering about the other component $\tau_y$. Following the same argument as above the torque $\tau_y$ contains the integral $\int xdx$, which vanishes so the $x$ component is the only surviving bit, so $\tau = IaB_y\hat{i}$. Using the magnetic moment $\mu$

$$\mu \times B = -Ia\hat{k} \times (B_x\hat{i} + B_y\hat{j}) = IaB_y\hat{i}$$

as above, so $\tau = \mu \times B$.

The net force of the loop

$$\int d\mathbf{F} = l \int d\ell \times \mathbf{B} = 0$$

since $\int d\ell = 0$ due to the loop being a loop.

(11) Bonus