**Solutions:**

(1) (a) By Gauss’s law for a cylindrical surface around the rod, the electric field is by \( \frac{\lambda}{2\pi \epsilon_0 r} \). This is the result in the frame shown in figure 5.30. Boosting to the frame of the particle gives \( \gamma \frac{\lambda}{2\pi \epsilon_0 r} \), since the field is transverse and since \( r \) doesn’t change in the transformation. The force is larger in the particle’s rest frame. (This provides another way to solve this problem.)

(b) There is now a non-vanishing \( B \)-field but the force vanishes since we are in the rest frame of the particle. The electric field has increased due to length contraction and we obtain the same result as above, \( \gamma \frac{\lambda}{2\pi \epsilon_0 r} \), as we must.

(2) Done in book. However, your solutions need to be clear and complete.

(3) The current along the \( y \) axis (pointing out in this sketch) produces a \( B \)-field that circulates around the point \((x = 0, z = h)\) in the \( x-z \) plane. Due to the symmetric arrangement, at the instant shown in the sketch, when the center of the square loop crosses the \( z \) axis, we have some hope to compute the flux, and its change.

The \( B \)-field is

\[
B = \frac{\mu_0 I}{2\pi r}
\]

or, in these coordinates \( B = \frac{\mu_0 I}{2\pi \sqrt{h^2 + (b/2)^2}} \)

in magnitude at the edges of the loop. Only the \( z \) components contribute to the flux. They are

\[
B_z = \frac{\mu_0 I}{2\pi \sqrt{h^2 + (b/2)^2}} \frac{b/2}{r} = \frac{\mu_0 Ib}{4\pi (h^2 + (b/2)^2)}.
\]

At the trailing edge the \( B \)-field points down and to the right while at the leading edge it points up and to the right. If the loop moves the small distance \( vdt \) it loses a bit of downward flux on the trailing edge and gains a bit of upward flux on the leading edge. Both these increase the flux up through the loop. These have the same magnitude, thus

\[
\mathcal{E} = -\frac{d\Phi_B}{dt} = -2b vdt \frac{d}{dt} B_z = -\frac{\mu_0 B^2 v}{2\pi (h^2 + (b/2)^2)}.
\]
The minus sign is Lenz’s sign and tells us that the induced current will run CW as viewed from above, creating an induced B-field that points downward. (I know that the question asks for magnitude; I just wish it didn’t.)

(4) (a) As we saw in class the area increases as \(bvdt\) so the rate of increase is \(bv\). Since the field is uniform, the induced EMF is \(Bbv\) and the current is \(I = Bbv/R\) (also as we found in class). The \(I\ell \times B\) force is simply \(F = I\ell B = B^2b^2v/R\). This force opposes the motion. (Check this by choosing a direction for \(B\), say up. Then, \(I\) runs down and \(I\ell \times B\) points to the left.) So,

\[
F = \frac{dp}{dt} \implies m\frac{dv}{dt} = -\frac{B^2b^2}{R}v
\]

This is a pretty easy differential equation to solve. It separates and is integrable.

\[
\int_{v_i}^v \frac{dv'}{v'} = -\frac{B^2b^2}{mR} \int_0^t dt' \implies \ln\left(\frac{v}{v_i}\right) = -\frac{B^2b^2}{mR}t
\]

So

\[
v = v_i e^{-\alpha t} \text{ with } \alpha = \frac{B^2b^2}{mR}
\]

It slows exponentially quickly.

(b) Let’s integrate up to find the distance,

\[
x = \int_0^\infty v(t)dt = \int_0^\infty v_i e^{-\alpha t}dt = -v_i/\alpha e^{-\alpha t}\bigg|_0^\infty = \frac{v_imR}{B^2b^2}.
\]

(c) The initial kinetic energy is \(1/2mv_i^2\) (if non-relativistic). This energy must end up as heat of the resistor. Let’s check. The power dissipated is \(P = I^2R\) and the current is \(I = Bbv/R = Bbv_i e^{-\alpha t}/R\) so the total energy is

\[
\int_0^\infty I^2R dt = \frac{B^2b^2v_i^2}{R} \int_0^\infty e^{-2\alpha t} dt = \frac{B^2b^2v_i^2}{2R} \frac{mR}{2B^2b^2} = \frac{1}{2}mv_i^2
\]

Neat! It works!

(5) (a) The magnetic field inside the solenoid is

\[
B(t) = \mu_o n I(t) = \mu_o n I_o \cos \omega t.
\]

Using Faraday’s law for this uniform field gives

\[
\mathcal{E} = -\pi r^2 \frac{dB}{dt} = \pi r^2 \mu_o n I_o \omega \sin \omega t \text{ in the CCW direction.}
\]

(b) The force on a bit of the ring is \(d\mathbf{F} = I d\ell \times \mathbf{B}\). The total force is directed radially outward (when positive) and

\[
d\mathbf{F}(t) = \frac{\pi r^2 \mu_o^2 n I_o^2 \omega d\ell}{R} \sin \omega t \cos \omega t.
\]

The force is largest when \(\omega t = \pi/4 + n\pi\) (outward) and \(\omega t = 3\pi/4 + n\pi\) (inward).

(c) Due to the radial nature of the force, it only serves to stretch or shrink the loop. If made out of stiff wire, the ring will not deform.
(6) We can use the prior result for a $B$-field on the axis of a ring, $b$ from the center,

$$B = \frac{\mu_0 I a^2}{2(a^2 + b^2)^{3/2}}.$$  

For really big $b$ ($b \gg a$) then

$$B = \frac{\mu_0 I a^2}{2b^3}.$$  

This is constant of over the other ring (also of radius $a$) so the magnetic flux is

$$\Phi_B = \pi a^2 \frac{\mu_0 I a^2}{2b^3}$$  

and the mutual inductance is $M = \frac{\Phi}{I} = \frac{\mu_0 \pi a^4}{2b^3}$.

(7) We did the first part of this one in class. The self-inductance is

$$L = \frac{\mu_0 \pi r^2 N^2}{\ell} \simeq 7.1 \times 10^{-3} \text{ H.}$$

Now, of course the field is not uniform in any finite length solenoid. The field lines diverge at the ends of the solenoid. To with a smaller actual $B$-field, we have over estimated $L$. From our experience last week, we might expect that the error to be roughly the radius divided by the length, which is about 2% here. There are tables of inductances of cylindrical coils.

(8) Postponed
(9) Postponed
(10) (worth 1 pt extra) Question on the colloquium