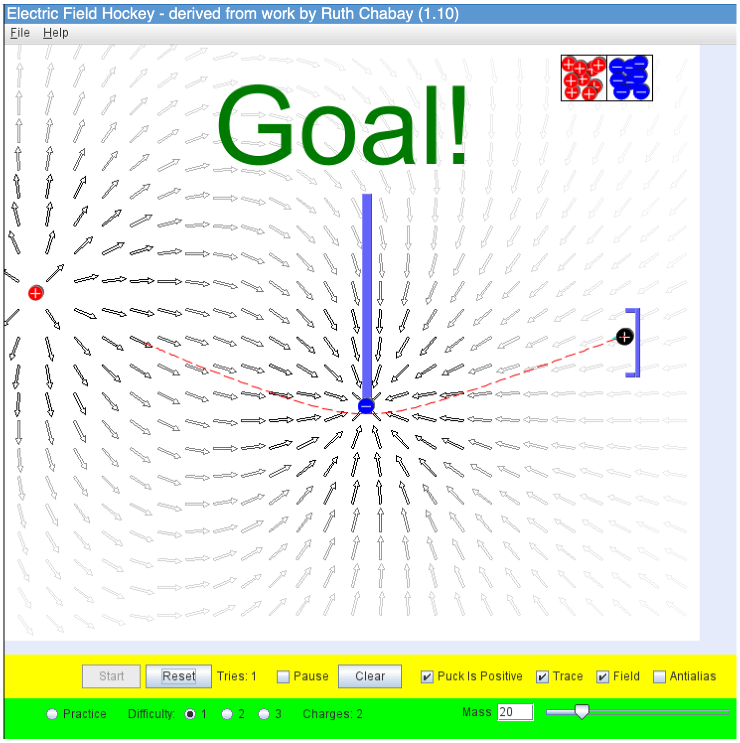
Solutions:

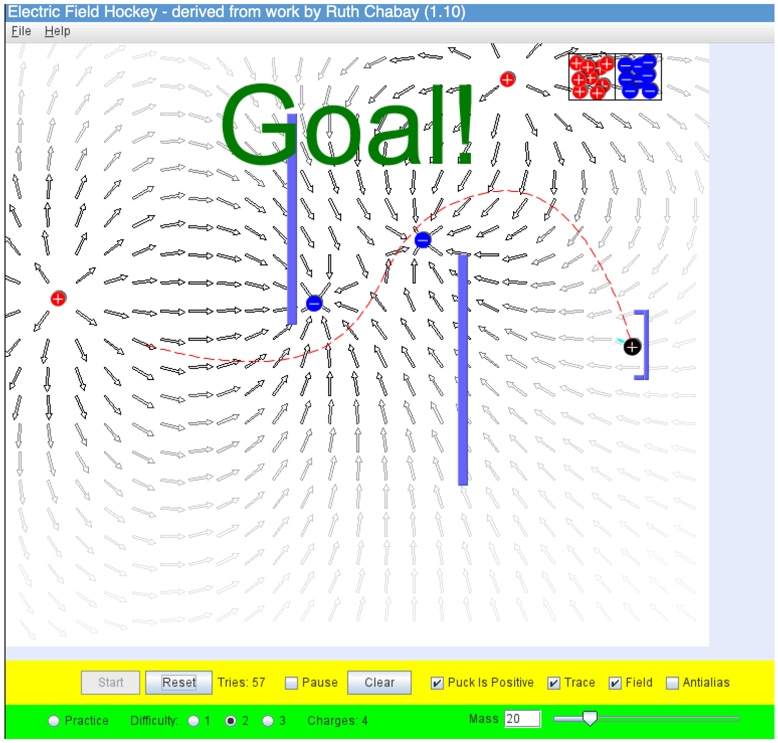
(1) Electric hockey: The masses should all be set at 20. The pictures should include fields and traces.

(a) The first level can be solved with two charges, a dipole. For instance,



The + charge acts as a stick and the - charge curves the trajectory to end up in the goal, much like what is done with spacecraft in planetary flybys. Only 2 charge configurations (dipole) receive full credit.

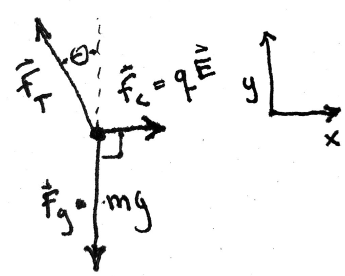
(b) Elegance tries my patience in level 2. While I had a seemingly nice 3 charge solution started, the fine tuning was lengthy so I went for a 4 charge version.



(2) A pendulum in an electric field

(a) The electric field points to the left so negative source charges are on the left and positive source charges are on the right. Since the pendulum is attracted to the right side of the picture it must have a negative charge.

(b) The charged pendulum is in equilibrium. The free body diagram (FBD) for this configuration is



so in the x-direction we have

$$\sum F_x = 0 \implies -F_T \sin\theta + qE = 0$$

Similarly, in the y-direction

$$\sum F_y = 0 \implies F_T \cos\theta - mg = 0.$$

Dividing these two and solving for the charge gives

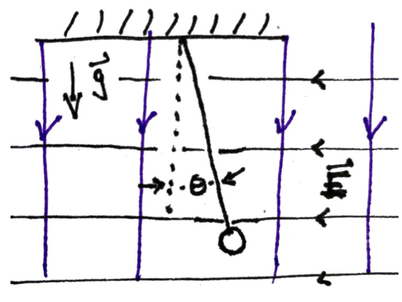
$$q = \frac{mg}{E}\tan\theta$$

(c) For the numbers given

$$q \simeq -1.4 \times 10^{-3}\ \mathrm{C},$$

accounting for the sign from part (a).

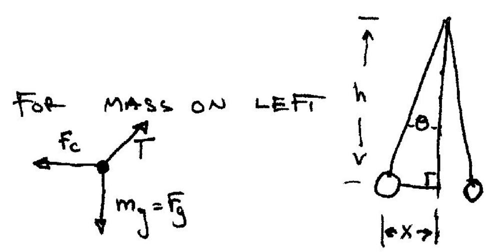
(d) Add the (uniform) gravitational field:



(3) This problem is solved in the book.

(4) This problem is also solved in the book, although you do not need to substitute numbers as done in the last step; David Morin did that just for fun. Notice the expansion of the integrand. It is the same as the one we used in the multipole expansion of the dipole field. Also you can see David's variation on our expression from Phys 195 $(1+x)^n \simeq 1 + nx$.

(5) This is a statics problem using Coulomb's force F_c. With the geometry diagram and the FBD we can obtain obtain the condition for equilibrium.



The FBD gives us two equations,

In the y direction $mg = T\cos\theta$ and, in the x direction $F_c = T\sin\theta$.

Dividing the second equation by the first gives

$$F_c = mg\tan\theta \implies \frac{1}{4\pi\epsilon_o}\frac{q^2}{(2x)^2} = mg\tan\theta = mg\frac{x}{h}$$

so that

$$q = \sqrt{\frac{16\pi\epsilon_o mgx^3}{h}} \simeq 2.86 \times 10^{-6} \simeq 3 \times 10^{-6}\ \mathrm{C}$$

(There is only 1 sig fig in the problem.)

(6) For the configuration two positive Q charges equidistant from a positive charge q, the force is zero since each charge Q exerts a force of the same magnitude but opposite direction on the charge q. Now suppose the charge q is bumped a wee bit away from equilibrium so that $x > 0$ and $x \ll \ell$. Now the net force is

$$F = \frac{q}{4\pi\epsilon_o}\left(\frac{Q}{(\ell+x)^2} - \frac{Q}{(\ell-x)^2}\right).$$

Since x is small compared to ℓ we can expand using $(1+x)^n \simeq 1+nx$. Setting this up by pulling a factor of ℓ^2 out of the net force we have

$$F = \frac{q}{4\pi\epsilon_o}\left[\frac{Q}{\ell^2}\left(1+\frac{x}{\ell}\right)^{-2} - \frac{Q}{\ell^2}\left(1-\frac{x}{\ell}\right)^{-2}\right].$$

Expanding we have,

$$F \simeq \left(\frac{q}{4\pi\epsilon_o}\right)\left(\frac{Q}{\ell^2}\right)\left(1-2\frac{x}{\ell} - \left(1+2\frac{x}{\ell}\right)\right) = -\frac{qQ}{\pi\epsilon_o\ell^3}x$$

Here the key idea is that the force is of the form $F = -k_{eff}\,x$ ($k_{eff} > 0$) so the same as a spring. Thus, the angular frequency is

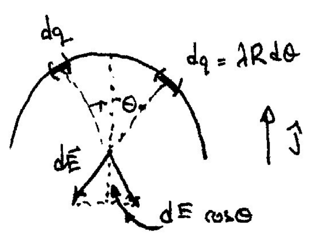
$$\omega = \sqrt{\frac{k_{eff}}{m}} = \sqrt{\frac{qQ}{\pi\epsilon_o m\ell^3}}.$$

(If it is not so familiar, the relation I used here is "$F = ma$" in the form

$$-\frac{qQ}{\pi\epsilon_o\ell^3}x = ma \implies \frac{d^2x}{dt^2} + \frac{qQ}{\pi\epsilon_o m\ell^3}x = 0.)$$

To solve this one you can also work from the expression for the energy $U(x)$ and find the derivative of the force.

(7) To compute the $\vec{E}$-field it is handy to consider the symmetry of the system.



From the diagram we can see that the horizontal components of $d\mathbf{E}$ from each half cancel and the vertical components add. So with

$$d\mathbf{E} = \frac{1}{4\pi\epsilon_o}\frac{\lambda R d\theta}{R^2}\hat{r}$$

from one bit of charge dq, we take twice the vertical component, $d\mathbf{E}\cos\theta$, and integrate over θ,

$$\mathbf{E} = \int_0^{\frac{\pi}{2}} \frac{1}{4\pi\epsilon_o}\frac{2\cos\theta\lambda}{R}d\theta(-\hat{\jmath}).$$

This is an integration of cosine and so we obtain,

$$\mathbf{E} = \frac{1}{2\pi\epsilon_o}\frac{\lambda}{R}(-\hat{\jmath}) = \frac{Q}{2\pi^2\epsilon_o R^2}(-\hat{\jmath})$$

where in the last step I used the total charge

$$Q = \int_0^\pi \lambda R d\theta = \lambda R \pi.$$

(8) From class, the magnitude of the electric field on the axis of symmetry is

$$|\vec{E}| = \frac{Qz}{4\pi\epsilon_o(z^2+b^2)^{3/2}}.$$

Differentiating to find the extrema we have

$$\frac{1}{(z^2+b^2)^{3/2}} - \frac{3}{2}\frac{z\cdot 2z}{(z^2+b^2)^{5/2}} = 0$$

Multiplying through by $(z^2+b^2)^{5/2}$ gives

$$z^2+b^2-3z^2 = 0 \implies z = \pm\frac{b}{\sqrt{2}}$$

Well that's the math. But are these roots the maximum or minimum? Since the electric field vanishes at infinity *and* at $z=0$, these must be maxima. The problem asks for the positive root.