Electromagnetism (PHYS 295): Mid-term Example

Spring 2024 v1.0

This example set of problems contains problems that could appear on the mid-term. Although it has more problems than will appear on the mid-term, it does not contain a complete listing of the types of problems that might appear on the mid-term. Aside from trying these problems, please review the problem sets, class notes, and your text. Please memorize Gauss' law in both differential and integral forms The equation sheet is on the second page. If you see something that's missing please let me know! Plan on bringing a working calculator.

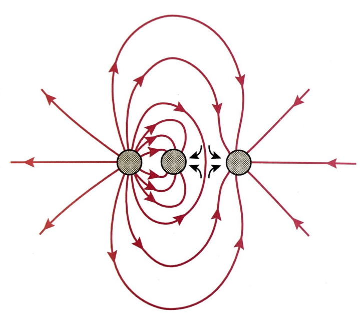
Topics include:

- Coulomb's law
- The relations among ρ, V, E, and F.
- Use of flux and Gauss' law to find the electric field
- Definitions and use of the energy U
- Visualization of electric field $\mathbf{E}$ with field diagrams and the electric potential V with equipotentials
- Dipoles and the multipole expansion
- Properties of conductors and insulators
- Use of image charges to find fields and charge densities.
- Capacitors, resistors and simple circuits

Our mid-term will be on Friday, March 1 during class time. It will be a 50 miniute, closed-book exam.

Example Problems:

(1) Here's a sketch of an electric field created by three charges:



Determine the signs of the charges and their relative magnitudes. (The black curvy lines show the direction of field lines at that point. Why do you think the author decided to include them?)

(2) You are given two charges of $+4q$ each and one charge of -1q.

(a) How would you place the three charges so that the net forces vanish on all of the charges?

(b) Using a full page of paper carefully sketch the electric field of your configuration.

(c) (Optional bonus) What is the nature of this equilibrium (stable, neutral, or unstable)? Assume that the two $+4q$ charges are fixed. Investigate this quantitatively. Hint: Try displacing the charge in two different directions. Alternately do a Taylor expansion around the equilibrium point.

(3) Starting from the differential form of Gauss' law derive the integral form.

(4) Consider a solid sphere is charged with radially dependent charge density $\rho = k_o r^2$ where k_o is a positive constant in units of C/m^5. Find the **E**-field inside and outside the sphere. Sketch the field.

(5) Consider a uniformly charged, long cylinder of radius R. Assume the volume charge density is ρ.
   (a) Using Gauss' law find the electric field inside ($0 \leq r \leq R$) and outside ($r \geq R$).
   (b) Sketch the electric field lines.
   (c) Make a plot of $|\mathbf{E}(r)|$ from $r = 0$ to $r = 4R$.

(6) An infinite sheet of charge has a uniform surface charge density σ. Find the **E**-field everywhere. Solve for the **E**-field around two infinite sheets of opposite charge separated by a distance d.

(7) Find the electric potential $V(z)$ on axis of a circle with uniform linear charge density λ and radius a. Find an expression for the field when $z > a$ and check that it conforms to what you expect.

(8) Find the electric field of a charge q positioned a distance h above a a conducting plate. Find the surface charge distribution on the plate.

(9) A charge q sits a bit off center by a distance a in a parallel plate capacitor with separation d ($a < d/2$). Find an approximate form of electric potential in the capacitor. Make a careful sketch of the electric field inside.

(10) A hollow spherical shell of radius a has charge Q, which is uniformly painted over the surface. Find the electric field inside and outside. By finding the energy stored in the electric field, find the energy stored in this configuration.

(11) Find the momopole, dipole (and quadrupole if you wish) moments of your configuration of charges in problem 2.

(12) A battery with potential $\mathcal{E}$, capacitor with capacitance C, and resistor with resistance R are connected in series around a loop. Initially the there is no charge on the capacitor. What is the charge at time t?

Handy Relations:

General:

$$\epsilon_o = 8.85 \times 10^{-12}\ \mathrm{C^2/Nm^2}$$
$$c^2 = a^2 + b^2 - 2ab\cos\theta$$
$$\text{For spheres } Area = 4\pi r^2 \text{ and } Vol = \frac{4\pi}{3} r^3$$
$$d\vec{a} = r^2 \sin\theta d\theta\, d\varphi\, \hat{r} \quad dv = r^2 \sin\theta d\theta\, d\varphi\, dr$$
$$F_x = -\frac{\partial U}{\partial x}$$

The Taylor series of a function $f(x)$ around $x = 0$ is

$$f(x) = f(0) + \left.\frac{df}{dx}\right|_{x=0} x + \frac{1}{2}\left.\frac{d^2 f}{dx^2}\right|_{x=0} x^2 + \frac{1}{6}\left.\frac{d^3 f}{dx^3}\right|_{x=0} x^3 + \ldots$$
$$(1+x)^n \simeq 1 + nx$$
$$\nabla = \hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z} \text{ in Cartesian coordinates}$$
$$\text{Laplace's equation } \nabla^2 V = 0$$

Legendre Polynominals

$$P_0(x) = 1,\ P_1(x) = x,\ P_2(x) = \frac{1}{2}(3x^2 - 1),\ P_3(x) = \frac{1}{2}(5x^3 - 3x), \ldots$$

E and **B** Fields:

$$\mathbf{E} = \frac{\mathbf{F}}{q} \text{ and } \mathbf{E} = -\nabla V$$
$$\mathbf{E} = \frac{1}{4\pi\epsilon_o}\frac{Q}{r^2}\hat{r}$$
$$U = \frac{\epsilon_o}{2}\int E^2 dv$$
$$V = \frac{1}{4\pi\epsilon_o}\int \frac{\rho dv}{r}$$
$$p = qd \text{ or more generally } \vec{\rho} = \int \vec{r}\rho dv$$
$$V_d = \frac{p\cos\theta}{4\pi\epsilon_o r^2}$$
$$V = \frac{1}{4\pi\epsilon_o}\left[\frac{1}{r}\int \rho dv' + \frac{1}{r^2}\int r'\rho P_1(\cos\theta) dv' + \frac{1}{r^3}\int r'^2 \rho P_2(\cos\theta) dv' + \ldots\right]$$
$$I = \int \mathbf{J}\cdot d\mathbf{a} \text{ and } \nabla\cdot\mathbf{J} + \frac{\partial\rho}{\partial t} = 0$$
$$Q = CV,\ V = IR,\ P = IV,\ C = \frac{\epsilon_o A}{d}$$
$$V = \frac{1}{4\pi\epsilon_o}\frac{q}{r} \text{ and } V_b - V_a = -\int_a^b \mathbf{E}\cdot d\mathbf{s}$$