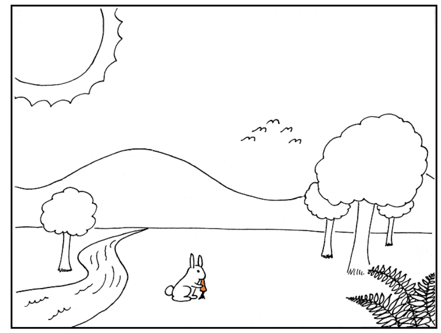
Topics in Mathematical Physics (PHYS 320)

Spring 2019 v5.5



In this course we review, learn, and practice a number of mathematical tools that have direct practical use in physics and related fields. These include ordinary differential equations, partial differential equations, special functions, and integral transforms. The idea is that these tools will be of use, not only in your remaining studies at Hamilton, but also in physics (or engineering, applied math, ...) graduate school. A "pedestrian guide" to the mathematics, the course focuses on the implementation of the methods and applications rather than the deep mathematical structure (Well, mostly. We will discover and prove aspects of a wonderful connection between solutions of differential equations and vector spaces.) So, even if you have had a course in one or more of the topics, the emphasis and even some of the methods will be new. I hope that you will master much of the material so that you can easily use in research, further studies, or elsewhere in life. Another goal of the course is to develop your skills in learning a new method and employing it. For no matter how many topics we discuss, there will always be more! Since you may need to learn one or more of these on your own, the skill of learning new methods is worth practicing.

I assume that you have had, and are comfortable with, linear algebra. If this material is rusty, Boas has a good bit on linear algebra as well - see chapter 3 sections 6 - 11.

Contact Info:

Seth Major
Science Center G052
x4919
smajor@hamilton.edu

My office hours will normally be 2 - 4 on Mondays. (I have Academic Council meetings from 4 to 6.)

Course Info:

The Phys 320 web site is
http://academics.hamilton.edu/physics/smajor/Courses/320.html
(a clickable link in the pdf version) or google "320 Hamilton's math methods"

Course objectives:

- To learn and become proficient at solving mathematical problems in physics.
  My expectation is that you will -
  - become fluent in the methods that you have used before (e.g. vector analysis, solutions of the ODE for simple harmonic oscillators),
  - become familiar to fluent with a number of new analytic methods (e.g. special functions, Laplace transforms, groups) and
  - acquire a much broader mathematical base from which to draw in the future – mathematical methods for which you have "active familiarity").
- To learn how to *learn* new mathematical methods for physics from a text.
- To become comfortable working with symbolic manipulators such as maple or mathematica.

Course Structure:

The course will normally be in our usual lecture/discussion format. I strongly encourage you to ask questions and make observations when these occur to you. Discussion is welcome!

The written work format will be somewhat unusual in that you may first encounter the material in your reading and in a small number of problems. In class we will focus on filling out understanding, answering questions, embellishing the material, and working through more examples.

In special functions section you will choose your favorite (or near favorite) to present.

We will also have an ODE workshop that will meet in a lab and use experiments!

Textbooks:

We will mainly draw from Mary Boas, *Mathematical Methods in the Physical Sciences*, 3rd edition. There are older editions in the Common Room.

There are a number of other texts which we will also refer to including:

- Potter and Goldberg (henceforth "PG"), *Mathematical Methods*
- Wong, *Introduction to Mathematical Physics*;
- Arfken and Weber, *Essential Mathematical Methods for Physicists* (known as "Baby Arfken"); Arfken, *Mathematical Methods for Physicists* (QA37.2.A74 1970) .
- Mathews and Walker, *Mathematical Methods of Physics.*

These are listed in order of increasing difficulty; Mathews and Walker is a graduate level text.

Homework - Solutions - Quizzes:

- *Daily Solutions*: On most class days, there be 2 or 3 solutions due in class. These will be relatively quick exercises on the reading. When you come to class hand in a copy and keep a copy for discussion. On these problems, I encourage you to work together. A grader will look these over and write comments. Complete "Daily Solutions", even if they contain some errors, to receive full credit.

- *Quiz Problem Sets*: At suitable intervals - I'm planning on about 4 during the semester - we will have a problem set. They are your own work - please do not collaborate on these - open textbook and notes, but not "open any resource". You may use Maple or Mathematica. Unlike traditional problem sets these must be entirely your own work. They will typically be released into the wild on Thursdays and be due on Tuesdays.

- *Solutions* to the daily problems will be either discussed or distributed during class.

Grades: Your semester grade will be determined by the following scheme:

Daily solutions	35%
Problem Sets	40%
Final	25%

Topics:

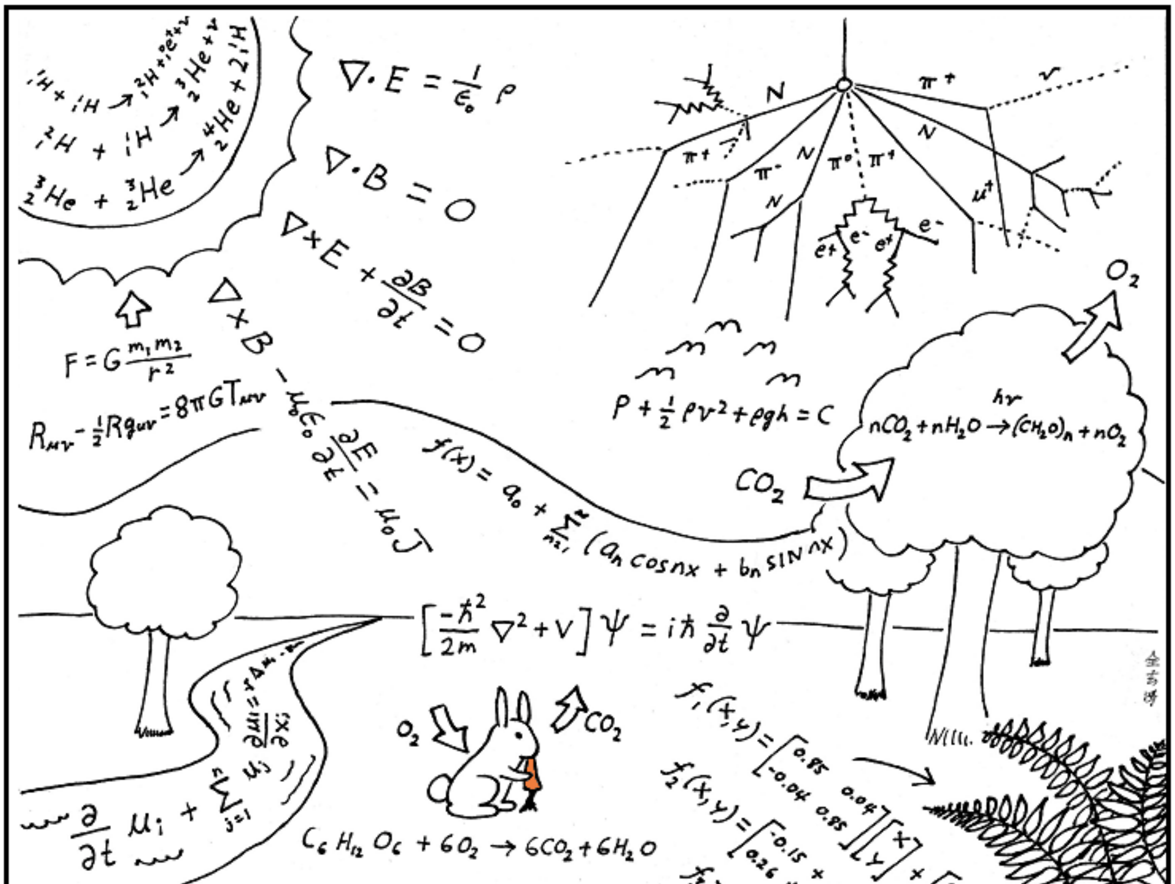
We will (un)cover *some* of these...

- **Complex numbers**: If there is a need
- **Methods of solving Ordinary Diff. Equ's (ODEs)**: first order, second order with constant coefficients, series method
- **Sturm-Liouville Theory**: Solution spaces as vector spaces
- **Dirac Notation and Solution spaces**: Dirac notation (QM)
- **Special Functions**: Laguerre, Legendre, Hermite, Bessels, Error, Dirac δ-function, ...
- **Fourier Series**: a theorem, computation of coefficients
- **Integral Transforms**: Laplace and Fourier
- **Partial Diff Equ's (PDEs)**: waves, diffusion, Schrödinger, Laplace, separation of variables
- **Groups**: Intro, examples, rotations in 3-space, representations, applications
- **Techniques of Complex Analysis**: analytic functions, integration, integral theorems

- **Tensors**: Definition and applications

We will not have time to discuss all of these topics. In the recent past we have chosen one of the last three topics. I expect that we will do this again after break.

A theme may emerge that is a common thread through most of the course...



This is how ~~scientists~~ math methods students see the world.

with apologies to the Abstract Goose