* INITIAL CONDITIONS FOR THE LENGTHENING PENDULUM

THE GENERAL SOL'N IS

$$\Theta(u) = A u^{-1} J_1(u) + B u^{-1} N_1(u) \qquad (18.6)$$

WITH $u = \frac{2\sqrt{g}}{v}\sqrt{l}$. KEEPING IN MIND THAT $l \propto t$ THEN AT $t=0$, $u = u_0 = \frac{2\sqrt{g}}{v}\sqrt{l_0} \neq 0$! ALSO NOTE THAT

$$\frac{d\theta}{dt} = \frac{d\theta}{du}\frac{du}{dl}\frac{dl}{dt} = \frac{d\theta}{du}\frac{\sqrt{2}}{\sqrt{l}}$$

SO WHEN $\frac{d\theta}{dt} = 0$ ~~SO~~ $\frac{d\theta}{du} = 0$. THE INITIAL CONDITIONS $\theta(0) = \theta_0$, $\dot{\theta}(0) = 0$ BECOME

$\Theta(u_0) = \theta_0$ AND $\frac{d\Theta}{du}(u_0) = 0$. NOW

$$\frac{d\theta}{du} = -\left[A u^{-1} J_2 + B u^{-1} N_2\right]$$

SINCE $\left(\frac{1}{z}\frac{d}{dz}\right)\left(z^{-1}\mathcal{C}_1\right) = -z^{-2}\mathcal{C}_2$ [FROM A&S 9.1.30] FOR ANY "BESSELY" FUNCTION $\mathcal{C}_n$.

THEREFORE THE IC'S GIVE

$$\begin{cases} A u_0^{-1} J_1(u_0) + B u_0^{-1} N_1(u_0) = \theta_0 \\ A u_0^{-1} J_2(u_0) + B u_0^{-1} N_2(u_0) = 0 \end{cases}$$

THESE CAN BE SOLVED USING LINEAR ALGEBRA GIVING

$$A = \frac{u_0^{-1} N_2 \theta_0}{u_0^{-1}\left[J_1 N_2 - J_2 N_1\right]} \quad \text{AND} \quad B = \frac{-J_2 u_0^{-1}\theta_0}{u_0^{-1}\left[J_1 N_2 - J_2 N_1\right]}$$

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BUT $u_0^{-1}\left[J_1 N_2 - N_1 J_2\right] = u_0^{-1}\left(\frac{-2}{\pi u_0}\right) = \frac{-2}{\pi u_0^2}$

$$\Rightarrow A = -\frac{\pi}{2} u_0 N_2(u_0)\theta_0 \quad \text{AND} \quad B = \frac{\pi}{2} u_0 J_2(u_0)\theta_0$$