Topics in Mathematical Physics (PHYS 320): PS 1

Spring 2019 v1.5

Welcome to the first problem set, primarily on ODE's!

- Please submit your solutions in class on Tuesday February 12.
- Please use your notes, Mathematica, Wolfram Alpha, and Boas, but no other resources. When you use Boas please *cite any references* (page number and formula number). Include printouts of your Mathematica notebook(s), as appropriate.
- Your solutions must be entirely your own work.
- Unlike the daily solutions I ask that you do not work with others, but please ask me questions if a question is unclear, if there is confusion, or if you are unsure how to proceed.
- **Please check your solutions.**

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(1) Consider the ordinary differential equation

$$\frac{du}{dx} + 2xu^2 = 0$$

(a) Plot the slope field on a domain of $(-2, 2)$.
(b) Find specific solutions to the differential equation when $y(-1) = 0.6$ and $y(-1) = -1$.
(c) Plot these solutions and your slope field in one plot.
(d) Is there a general solution valid everywhere? If so explain why. If not write a solution that doesn't belong in the family of solutions you have found so far. Finally, comment on the nature of the solutions to this ODE.

(2) Suppose you are investigating an air handling system for a concert hall. At elevated CO_2 levels the audience feels drowsy, which is crummy for audience and performers alike!

On average a seated person takes 18 breaths per minute, each breath exhales 0.016 m^3 of air with 4% CO_2. A concert with 600 people in the hall starts with a concentration of 0.10% CO_2. The ventilation system delivers 10 m^3 per minute of outside air to the 2100 m^3 room while removing the same amount of inside air. This outside air has the CO_2 concentration of 411 ppm = 0.0411% (which is the Jan 2019 Mauna Loa observation station concentration, in case you are curious.)

(a) Derive an ordinary differential equation describing the concentration of CO_2 in the concert hall.
(b) Does the the CO_2 concentration remain below the 'drowsy' level of 1000 ppm for the 3 hour concert?

(3) Consider

$$x^3 + u^3 - xu^2\frac{du}{dx} = 0$$

(a) Describe this ODE.
(b) Find the general solution to this differential equation, or if that is not possible, describe where your family of solutions is valid.

(4) Consider

$$y'' + 2y' + 10y = 26\sin(2x)$$

(a) Describe this equation.

(b) Solve the initial value problem with $y(0) = 1, y'(0) = 0$.

(c) To obtain the largest amplitude steady-state oscillation in $y(x)$ to what value should I change "2" to in $\sin(2x)$? An approximate answer is fine.

(5) Consider initial value problem

$$u'' - 4u' + 8u = 0,\ u(0) = 2,\ u'(0) = 0.$$

(a) Describe the equation.

(b) Find the solution.

(6) Solve the initial value problem:

$$(x+u)\frac{du}{dx} + u = x\,,\ u(1) = 0$$

(7) Solve the initial value problem $u'' + 2u' + 4u = 0,\ u(0) = 1, u'(0) = 2$.

(8) (a) Find a general solution of

$$u'' + u = e^{-x}$$

(b) Solve the initial value problem with $u(0) = 0$ and $u'(0) = 2$.

(9) The rate at which the temperature of an object changes is proportional to the difference between its temperature and the environment's temperature. Andrew brews a mug of coffee just as Phys 320 is starting, at 2:30 PM. The coffee is at 180^o. The classroom is at 70^o. After 5 minutes Andrew finds that the coffee has cooled to 160^o. When will it be at a pleasant 140 or cold 110^o?