Topics in Mathematical Physics (PHYS 320): PS 2 Spring 2017

Welcome to the problem set on Fourier series, Laplace transforms, and series solutions!

- Please submit your solutions by 5 PM, Friday March 10.
- Please use your notes Mathematica, Wolfram Alpha, Schaum's, and Boas, but no other resources. Include printouts of your work with these programs.
- Please *cite any references* (source, page number and formula number, as appropriate).
- You may not consult any other resources such as the internet.
- Your solutions must be entirely your own work.
- Please check your results.

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(1) Find the Fourier series of

$$f(t) = \begin{cases} -t, & -L \le t \le 0 \\ t, & 0 \le t \le L \end{cases}$$

(2) Find the Laplace transform of

$$f(t) = \sinh(3t)$$

(3) Use Laplace transforms to solve the initial value problem

$$2u'' + 50u = 100 \sin \omega t \text{ with } u'(0) = u(0) = 0$$

for all ω.

(4) (a) Use the Frobenius (series) method to find the general solution to

$$xu'' + (1 - \gamma)u' + u = 0.$$

in which γ is a non-negative constant.

(b) Note any subtleties you encountered. If you didn't notice any, do you have two independent solutions valid everywhere? If not, where are they valid?

(5) In class we solved the Schrödinger equation for the harmonic oscillator potential in 1D. In **3D** ($U(r) = 1/2\, k\, r^2$) one encounters the differential equation

$$L[u] = \frac{d^2u}{dr^2} + \kappa u - \beta^2 r^2 u - \frac{\ell(\ell+1)}{r^2} u = 0.$$

You may want to know something about the parameters β, κ, and ℓ. The first two are numbers, β sets the length scale of the problem and κ is proportional to the energy. The parameter ℓ is an integer.

(a) Describe the equation's singularities.

(b) Let's find solutions that satisfy the boundary conditions $u(0) = 0$ and

$$\lim_{r \to \infty} u(r) = 0.$$

(Why?) in **two** steps: First, rewrite the equation in the asymptotic limit, $r \to \infty$. To do this, only keep derivatives and terms large in the limit.

(c) Solve the resulting equation in asymptopia.

(d) Second, with this observation make a change of variables

$$u(r) = e^{-\beta r^2/2} z(r).$$

Show that the differential equation becomes

$$r^2 z'' - 2\beta r^3 z' + \left[(\kappa - \beta) r^2 - \ell(\ell + 1)\right] z = 0 \tag{1}$$

(e) Have a crack at solving Eq. (1). Hints: (1) If you encounter a couple of roots, think through the physics. One makes sense. One does not.