Topics in Mathematical Physics (PHYS 320): PS 3 Spring 2017

Welcome to the problem set on Sturm-louville theory, special functions, and a little PDEs!

- Please submit your solutions on class on Thursday April 27 - Oh dear I won't be here! Please come by with questions before Wednesday morning.
- Please use your notes Mathematica, Wolfram Alpha, Schaum's, and Boas, but no other resources. Include printouts of your work with these programs.
- Please *cite any references* (source, page number and formula number, as appropriate).
- You may not consult any other resources such as the internet.
- Your solutions must be entirely your own work.
- Please check your results.

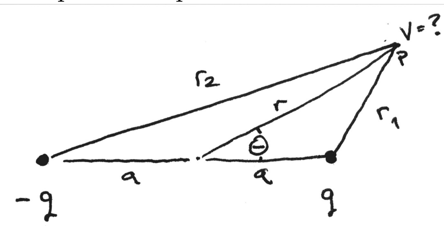
---

(1) (10 pts.) *Potentials*

(a) Two charges - equal but opposite - make a dipole. The potential is

$$V = \frac{q}{4\pi\epsilon_o}\left(\frac{1}{r_1} - \frac{1}{r_2}\right).$$

Using the geometry shown express this potential in terms of Legendre polynominals.



See Marcos' "Fun Facts" for the generating function. Simplify your expression as much as possible and derive the first *three* non-vanishing terms. Identify the 'far field' term - the field seen in asymptopia - and the 'near field' terms.

(b) Now consider a distribution with a +q charge at $-a$, a -2q charge at the origin, and a +q charge at a. What is this new potential? Express the potential in terms of Legendre polynominals. List the first two non-vanishing terms and explain the dependence on r relative to the dipole in the previous part.

In all cases assume that the potential vanishes at infinity.

(2) (10 pts.) Boas page 637 problem 3

(3) (15 pts.) In the theory of diffraction through a circular hole, you find that the amplitude A of the diffracted wave depends on the integral

$$A \sim \int_0^a J_o(br)r\,dr$$

where a is the radius of the circle and $b = 2\pi\sin(\alpha)/\lambda$. The angle α is shown in the figure and λ is the wavelength of the light.

(a) Using a recursion relation from Leo's "Amusing Facts" solve this integral by observing that the integrand is secretly a total derivative.

(b) Intensity is proportional to the square of the amplitude. By looking up the zeros of the Bessel function find the angles of the first 3 dark bands (zero intensity) for green light $\lambda = 5.50 \times 10^{-7}$ m. Assume the radius of the circle is 0.500 cm.

(c) **Extra:** (5 pts.) Arfken writes "Had this analysis been known in the 17th century, the arguments against the wave theory of light would have collapsed." Why? [BTW, this is the same analysis that gives the Rayleigh criterion for the resolution of two point sources

$$\Delta\theta = 1.22\frac{\lambda}{d}$$

$(d = 2a)$.]