

Topics in Mathematical Physics (PHYS 320): QPS 2 Spring 2009 v1.1

This is the “problem set quiz” on our study of ODEs. Please submit your solutions sometime before the end of Monday, March 9. You may consult Maple, Boas, your Phys 320 class notes, and standard references such as the Schaums. Please cite any references (source, page number and formula number as appropriate) as you use them. Do not consult the internet. Your solutions must be entirely your own work.

Ask questions when you have them. Try email: smajor@hamilton.edu or stop by my office. Enjoy!

- (1) Show that

$$3u^2u' + x^2 = 0$$

is exact and find a solution.

- (2) For the differential equation

$$\frac{dy}{dx} + \frac{xy^2 - 8x}{y} = 0$$

- (a) Describe the equation.
(b) Solve the initial value problem with $y(1) = 3$.
(c) Solve the initial value problem with $y(3) = 2\sqrt{2}$.
(d) Are these questions well-posed? If not, explain the nature of the space of solutions.

- (3) Consider

$$y'' + 2y' + 10y = 26 \sin 2x$$

- (a) Describe this equation.
(b) Solve the initial value problem with $y(0) = 1, y'(0) = 0$.
(c) To obtain the largest amplitude steady-state oscillation in $y(x)$ what should I change? To what value?

- (4) Consider initial value problem

$$u'' - 4u' + 8u = 0, u(0) = 2, u'(0) = 0.$$

- (a) Describe the equation.
(b) Find the solution.

- (5) Solve the initial value problem:

$$(x + u) \frac{du}{dx} + u = x, u(1) = 0$$

- (6) Solve the initial value problem $u'' + 2u' + 4u = 0, u(0) = 1, u'(0) = 2$.

- (7) Obtain the general solution of $u'' + 2u' + u = 0$.

- (8) (a) Find a general solution of

$$u'' + u = e^{-x}$$

- (b) Solve the initial value problem with $u(0) = 0$ and $u'(0) = 2$.

- (9) Find the Wronskian for the “spherical Bessel equation”

$$x^2y'' + 2xy' + (x^2 - \ell(\ell + 1))y = 0.$$

- (10) Consider the equation

$$\frac{du}{dx} = u \sin x.$$

- (a) Describe this equation.
(b) Find the general solution.

- (11) A 2 kg mass is suspended by a $k = 32$ N/m spring. A force of $0.1 \sin 4t$ is applied. Calculate the time required for failure to occur if the spring breaks when the amplitude exceeds 0.5 m. Neglect damping and assume the mass starts from rest.
- (12) A substance evaporates at a rate proportional to the exposed surface area. You shape a “Peeps” candy into a sphere with radius 1 cm. When your roommate discovers the candy six months later, the sphere has shrunk. It now has a radius of 0.8 cm (excluding the dust).
- How long will it take before the radius is 0.25 cm?
 - If you had left it in the original non-spherical shape would it evaporate more or less quickly? [I take no responsibility for the veracity of the stated Peep properties.]
- (13) The rate at which the temperature of an object changes is proportional to the difference between its temperature and the environment’s temperature. Andrew brews a mug of coffee just as Phys 320 is starting, at 2:30 PM. The coffee is at 190° . The classroom is at 70° . After 10 minutes Andrew finds that the coffee has cooled to 150° . When will it be at a pleasant 110° ?