Math Methods (PHYS 320):14

Spring 2017

Reading: For more on on Sturm-Louville theory Arkin and Weber posted on the 320 website
Boas on (your) special function (if none at least read Ch 12 section 22)

(1) Consider the Sturm-Louville eigenvalue problem

$$L \mid u\rangle + n(n+1) \mid u\rangle = 0$$

over the interval $-1 \leq x \leq 1$ with

$$L = (1-x^2)\frac{d^2}{dx^2} - 2x\frac{d}{dx}$$

.

(a) Is L self-adjoint? Explain.

(b) Are the solutions orthogonal? Explain.

Choose one of the following two problems to solve.

(2) A remarkable test of "local realism" in quantum mechanics, proposed by Hardy in 1993, was completed in 1999. In the experiment two particles pass through different detectors that have two settings and two possible outcomes for each setting. Let call the settings, or bases, a and b and the outcomes $+$ and $-$. A state of one particle can described by the kets $\mid a+\rangle$ or $\mid a-\rangle$ or $\mid b+\rangle$ or $\mid b-\rangle$. Assume that the $+$ and $-$ basis vectors are orthonormal in each basis. The state $\mid b+\rangle$ is related to the states in the a basis by

$$\mid b+\rangle = \cos\theta/2 \mid a+\rangle + \sin\theta/2 \mid a-\rangle.$$

In Dirac notation the states for more than one particle are "pasted" together. For example suppose that the first particle is in state $\mid a+\rangle_1$ and the second is in state $\mid b+\rangle_2$ then the state of the two is the product

$$\mid a+, b+\rangle = \mid a+\rangle_1 \mid b+\rangle_2$$

Quantum mechanics is unique among physical theories for allowing a superposition of such states.

The Hardy state is given by

$$\mid \psi\rangle = \cos^2\theta/2\,[\cos\theta/2 \mid a+, a+\rangle - \mid a+, a-\rangle - \mid a-, a+\rangle]$$

where $\theta \simeq 76.35^o$. Let's check some predictions arising from the state:

(a) Show that a system prepared in the Hardy state is never found in the state $\mid a-, a-\rangle$. You can do this by checking $\langle a-, a- \mid \psi\rangle = 0$.

(b) Show that a system prepared in the Hardy state is never found in the state $\mid a+, b+\rangle$.

(c) Show that a system prepared in the Hardy state is never found in the state $\mid b+, a+\rangle$.

(d) Show that a system prepared in the Hardy state is in the state $\mid b+, b+\rangle$ 9 % of the time, i.e. show that $|\langle b+, b+ \mid \psi\rangle|^2$ is approximately 0.09.

(e) EXTRA: Show that these results are incompatible with local realism. You can accomplish this by showing that the above results are impossible if you assign each particle properties for each setting. In the language of our speaker Don Spector last fall, there are no "tickets" you can give to the particles that lead to the experimentally-tested predictions above.

(3) Let's use the basis functions

$$\langle x \mid n \rangle = \frac{1}{\sqrt{\pi}} \cos(nx)$$

on $-\pi < x < \pi$. (n is an integer.) The inner product is

$$\langle u \mid v \rangle = \int_{-\pi}^{\pi} u^*(x) v(x) dx$$

so the weighting function is 1 and all these functions are real. Suppose you have a ket $\mid v\rangle$ which is the function

$$\langle x \mid v \rangle = 2\cos x + \cos(2x).$$

(a) Find $\langle 1 \mid v \rangle$.
(b) Find $\langle n \mid v \rangle$.