Classical Mechanics (PHYS 350): Guide 12 The Last Fall 2011 v1.0

We finish with our initial discussion of rigid body motion and start out brief excursion into coupled oscillations. This is a rich subject that has a simple question - what happens when a pair of sho's interact?

In class we will discuss this problem on Monday after Thanksgiving break, then we will return to rigid body mechanics - I realized that it would be best to have some problems on coupled oscillations on this guide so I changed the topic for Monday. The material is covered in T&M Chapter 12.

Reading:

T & M Chapter 11

T & M Chapter 12 (sections 1 - 6)

Problems: Due Wednesday November 30 at 5 PM.

(1) A frisbee is thrown into the air with a definite wobble. If air friction exerts a frictional torque $-c\omega$ on the rotation of the frisbee, show that the component of ω in the direction of the symmetry axis decreases exponentially in time. Show also that the angle between the symmetry axis and the angular velocity vector ω decreases in time if the moment of inertia around the symmetry axis is larger than the other moment of inertia. (A frisbee is a symmetric top.) Thus, the amount of wobble steadily diminishes if there is air friction.

(2) On tops

  (a) Write the kinetic energy of the symmetric top in terms of the Euler angles. Hint: Use the ω's in terms of the angles.

  (b) For a familiar symmetric spinning top the potential can be written as $U = MgR\cos\theta$. Including this in the Lagrangian, and using the techniques we discussed in the central potential problem, show that the effective potential is

$$U_{eff}(\theta) = \frac{(P_\phi - P_\psi \cos\theta)^2}{2I\sin^2\theta} + \frac{P_\theta^2}{2I_3} + MgR\cos\theta$$

  (c) Discuss the motion that follows from this effective potential.

(3) You stand on a cliff enjoying the view. Unfortunately someone behind you stumbles, reaches out, and pushes you in a direction perpendicular to the surface of the cliff. This gives you angular momentum. Do you necessarily fall off? Explain your answer in detail, including discussion of the torque(s), dynamics of your angular momentum, and inertia tensor. Remind me to mention a nice demo.

(4) Double Pendulum: Find the equations of motion for small oscillations. To do this you can make the small angle approximation at the level of the Lagrangian or at the level of the equations of motion. Find the angular frequencies of the normal modes. Make a sketch of these oscillations.

(5) Find the angular frequencies of the normal modes for the small angle "mass-on-a-ring" example from the Friday before break. Make a sketch of the modes.

(6) **Linear triatomic molecule** Consider a mechanical model of the CO_2 molecule where the three point-like atoms are confined to a line with the carbon at the center. They are attached by springs. Let the masses of the oxygen be m and the carbon be M. Finally let the spring constants be k.

  (a) How many degrees of freedom are there?

  (b) Make a choice for the coordinates and express the kinetic and potential energies in these coordinates.

  (c) Write the Lagrangian of the system.

(d) Find the normal mode angular frequencies - Wait! a moment of thought here could make the following calculation more expeditious. Have you made a good choice of coordinates to discuss the oscillations (and only the oscillations)? No matter what you conclude - no worries!

(e) Sketch the normal modes

The normal modes from this model agree quite well with observed absorption in the infrared.

Friday Class: Stay tuned...