Classical Mechanics (PHYS 350): Week 13

Fall 2005 v3.0

The dynamics of rigid bodies culminates our work on center of mass, relative motion, and dynamics in non-inertial frames. Using elements from the past three weeks we will investigate the dynamics of rotating and translating arbitrarily-shaped objects. Key new pieces of formalism are the **inertia tensor, principle axes, Eulerian angles, and Euler's equations**. We will spend Thanksgiving week and the week after on the subject.

Looking ahead, we will apply our new understanding of classical mechanics to understanding toys. Please have a look at the possible projects so you can choose one after the Thanksgiving holidays.

Reading:

Chapter 11 T & M pack a lot in this chapter. I encourage you to read carefully and slowly. (The Surgeon General says: Few activities promote the digestion of a holiday meal better than reading about the dynamics of rigid bodies.)

Problems:

These problems are due Friday December 2 at 5 PM. Some of these are involved; fortunately you have two weeks! Feel free to email questions before, during, or after break.

(1) A thin uniform rectangular plate (lamina) is of mass m and dimensions $2a$ by a. Choose a coordinate system such that the plate lies in the xy plane and has the origin at a corner. Find (a) the moments and products of inertia (b) the moment of inertia about the diagonal through the origin (c) the angular momentum about the origin when the lamina is spinning with angular velocity ω around the diagonal (d) the kinetic energy.

(2) A frisbee is thrown into the air with a definite wobble. If air friction exerts a frictional torque $-c\omega$ on the rotation of the frisbee, show that the component of ω in the direction of the symmetry axis decreases exponentially in time. Show also that the angle between the symmetry axis and the angular velocity vector ω decreases in time if the moment of inertia around the symmetry axis is larger than the other moment of inertia. (A frisbee is a symmetric top.) Thus, the amount of wobble steadily diminishes if there is air friction. **Demo**?

(3) 11-2 calculating moments of inertia

(4) 11-4 find the radius of gyration

(5) 11-6 dynamics as sleuth! Use a Lagrangian description. Note the operative instruction "in detail". Have you seen the **demo** which goes along with this?

(6) 11-9 a board on a cylinder - recall 2-42 and Greg's presentation...

(7) 11-11 How long does a cube balanced on edge take to fall on a side?

(8) 11-15 Kater's pendulum

(9) 11-20 a falling rod. Find the velocity of the end of the rod when it hits the ground. Compare this to the final velocity of a mass dropped from the center of mass height of the rod. **Demo!**

(10) 11-28 Very useful for analyzing a certain circular "toy" coming up...

(11) The Double Pendulum Returns: Numerically integrate the equations of motion for the double pendulum for two sets of initial conditions (one which differs from the other by 10^{-3} degrees) and make a plot of $\ln|\delta\phi|$ versus the dimensionless time τ. Use initial conditions $\dot{\phi}(0) = \dot{\theta}(0) = 0$, $\phi(0) = 0$, and $\theta(0) = 55, 65, 75, 85,$ and 90 degrees. Find the sign of the Lyapunov exponent λ for each of these cases. At about what angle does the system become chaotic?