Classical Mechanics (PHYS 350):Week 2

Fall 2013 v1.0

In Chapter 4 we delve into topics related to the study of periodic motion. Much of this material will be familiar although potentially new topics include phase space portraits, electrical analogs, coupled oscillations, and two dimensional oscillators.The main new analytical tool is phase space - a plot in momentum/position space. Sounds simple but proves to be very useful. We will use the review of the damped motion as an example of phase portraits.

Next week we continue our work on reviewing Newton's world by studying gravitation and rocket motion.

Reading:

Chapter 4
and we'll be moving into Chapter 5

Problems:

All problems are from Morin unless noted otherwise. Answers to problems ("P") are in the book. Answers to exercises ("E") are not in the book. Once you have gotten started on the problems and exercises please do not hesitate to ask questions. Your solutions are due in class on Wednesday. Office hours will be on Tuesday afternoon, since there are senior talks on Monday.

(1) The Lorentz force in 3D: (Assume all speeds are non-relativistic, $v << c$.) We'll align the z-axis with the magnetic field. Then $\vec{B} = B\hat{k}$ and assume that $\vec{E} = E_y\hat{j} + E_z\hat{k}$. This is the same field configuration as we discussed in class.
  (a) Find the equations of motion. (We did this in class.)
  (b) Solve for the motion in the z direction.
  (c) Show that the velocities in the x and y directions are periodic. Averaging over one period show that the average velocities, "$\langle\rangle$", satisfy

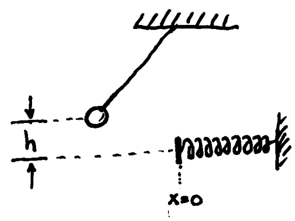
$$\langle \dot{x} \rangle = \frac{E_y}{B} \text{ and } \langle \dot{y} \rangle = 0.$$

  (d) Integrate the equations of motion with the initial conditions

$$x(0) = -A/\omega_c,\ \dot{x}(0) = E_y/B \text{ and } y(0) = 0\, \dot{y}(0) = A$$

  where ω_c is the cyclotron frequency we introduced in class. You have the parametric equations from last week. Plot the projections of the trajectories in the $x-y$ plane for the cases
    (i) $A > |E_y/B|$
    (ii) $A < |E_y/B|$
    (iii) $A = |E_y/B|$
  Feel free to use your plots from last week, just add in the physical interpretation.

(2) A pendulum bob of mass m is raised to a height h and released from rest. After hitting a spring with a non-linear restoring force, $F = -kx - bx^3$, the bob comes to rest at a location x_f. What is the value of x_f?



Hint-Work

(3) Suppose you drop a (smooth) beach ball (empty weight 119 g radius 19.0 cm) from the top of the Science Center. At what acceleration does it fall? How does the height fallen depend on time? Be complete. Compute the Reynold's number to determine the correct drag force to use. Be sure to include the effects of the air including buoyancy and drag.

(4) Phase Portraits: We've had a quick look at two phase portraits. In this problem we'll take another look at the phase portrait of simple harmonic motion
   (a) An oscillator has an amplitude of x_o and a natural angular frequency of ω, what is the (general) solution $x(t)$? Feel free to solve the equation of motion or simply write down the solution. Take the derivative so we have the velocity $\dot{x}(t)$.
   (b) To investigate the shape of the plot show that the solution satisfies

$$\frac{x(t)^2}{x_o^2} + \frac{\dot{x}(t)^2}{x_o^2\omega^2} = 1,$$

   which . . . is the equation for an ellipse. Sketch the phase portrait.
   (c) Find how much you would rescale the time axis so that the phase portrait is "unsquished", that is, a circle. Interpret the rescaling.

(5) Suppose you have a cylindrical rubber ducky jauntily bobbing in a bath. Assume that the ducky floats vertically so that its cross section is always the same area, πr^2. Show that the ducky's motion is simple harmonic motion. Find the period of oscillation. Please neglect the viscosity in the bath water.

(6) Two masses m_1 = 101 g and m_2 = 201 g glide frictionlessly on an air track. They are connected by a spring with spring constant k = 0.511 n/m. Find the frequency of oscillation for this system.

(7) 3.56 E

(8) 4.6 P

(9) 4.24 E

Friday Class: Something new on less linear oscillators