

This week we will be finishing oscillations with Fourier series and then moving onto to more of Newton’s world including gravitation and momentum conserving but mass changing processes known as “rocket problems”.

Ooops! I neglected to check my schedule before announcing our Friday schedule in class. I am gone *next* week rather than then the *following* week. Therefore we will have no class on Friday September 27. I will have office hours on Tuesday afternoon but will be leaving Wednesday afternoon after class.

Reading:

Chapter 5, a mixture of energy review and new stuff.

Problems: Due Wednesday in class.

All numbered problems are from Morin.

- (1) Consider a damped oscillator with natural angular frequency ω_o , damping β , and driving force with angular frequency ω . . As you know, it has an equation of motion of the form

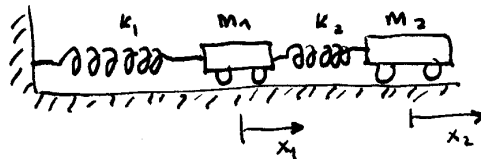
$$\ddot{x} + 2\beta\dot{x} + \omega_o^2 = A \cos(\omega t).$$

Show that the amplitude of the response is maximum when

$$\omega = \sqrt{\omega_o^2 - 2\beta^2}$$

Show also that this angular frequency is approximately ω_o for a high Q system. Find the next order correction to this using $(1 + x)^n \approx 1 + nx$.

- (2) You have a circuit built from an inductor (L), a capacitor C , and two resistors each with resistance R all connected one after the other in a closed loop. What is the equation of motion for the charge? If the system starts with charge at rest on the capacitor, what is the solution of the equation of motion? Assume that $\sqrt{L/C}(1/R)$ is small.
- (3) You place a “simple” (that means, “small angle approx is fine”) pendulum of length ℓ in a fluid with damping force $2m\sqrt{g\ell}\dot{\theta}$. Find the equation of motion and the general solution for the resulting motion.
- (4) 4.9 P
- (5) For this system of springs and carts



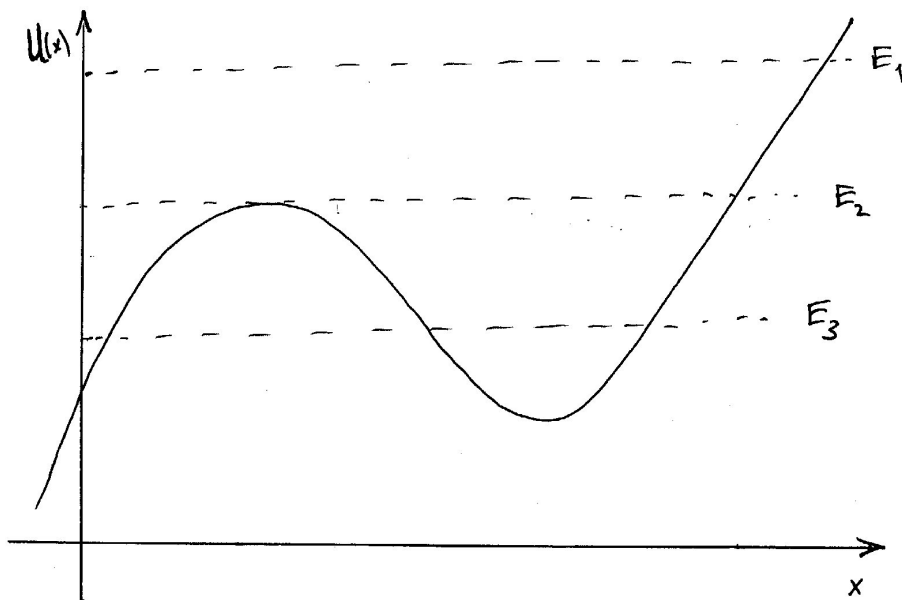
with $m_1 = m_2$, $k_1 = 3k$ and $k_2 = 2k$, find (a) the angular frequencies of the normal modes and (b) the description of the motion for each mode. Please neglect friction.

- (6) Phase Portraits:
 - (a) As done in class for the underdamped case, find the phase portrait for critically damped motion when the initial displacement and velocity are positive.
 - (b) Trajectories in phase space are contours of constant energy. Draw a phase portrait for the unstable motion described by

$$\ddot{x} - x = 0.$$

Is energy conserved? What type of curve are you drawing?

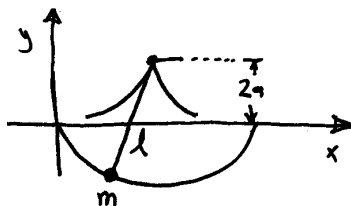
- (c) Looking at the potential $U(x)$ sketched here, identify the region(s) of oscillatory motion. Draw the phase portrait along the trajectories (including direction) in phase space at E_1 , E_2 , and E_3 . Also identify the stable and unstable equilibrium points.



- (7) (Problem for Oct 4) As sailing and navigation developed the technology of keeping time failed to keep up with it, with disastrous consequences (see). Pendulum clocks don't work very well since the period of the clock depends on angular amplitude (see equation 4.61 in Morin); they are not isochronous. One way to make a better pendulum clock is to shorten the string as it swings. (Huygens (yup, same one) is given credit for working this out.) This "shortening" can be accomplished by installing curved metal "chops" in the shape of a cycloid. You can see them in many clocks today. These chops cause the pendulum mass to follow a cycloid with trajectory given by

$$x = a(\phi - \sin \phi), \text{ and } y = a(\cos \phi - 1)$$

with the length of the pendulum given by $\ell = 4a$. Compared to last week these are shifted and inverted so that the curves look like:



Show that this pendulum is exactly isochronous with natural angular frequency $\omega_o = \sqrt{g/\ell}$.

Friday (October 4) class: Problems - Solutions - Demos

We will select these projects on Wednesday so please have a look at them before then.

There will be teams of 2 to work on a problem and present, or run the demo, if there is one. I suspect that we will not have time for more than 6-7 10 minute presentations on Friday. To save time and to focus on the physics highlight only the tricky steps in your presentation, work no routine algebra on

the board. Instead, if there is a longer calculation then make a handout of the solution (and please bring 17 copies).

- (1) **An Isochronous Pendulum** “1673 Nobel Prize” material! (if a Nobel Prize had existed in those days) Present the definition of a cycloid, the solution, and the historical context. For this last point an interesting telling may be found in Dava Sobel’s “Longitude.” Check out the beginning of the book, pages 1 - 40. Include a description of the problem and dead reckoning.
- (2) **Falling Chain: Part I** Once there was this physics problem, and its associated solution, that was passed down from textbook to textbook for many “generations”, about 150 years. The only problem was - the solution was wrong...
 You hold each end of a long chain. You release one end of the chain. What is the tension as a function of distance fallen? Use forces to solve for the tension. Seems innocent enough.
 Convince the assembled classical mechanics that *your* solution is *the* correct one!
- (3) **Falling Chain: Part II** You hold each end of a long chain. You release one end of the chain. What is the tension as a function of distance fallen? Use energy conservation to solve for the tension. Seems innocent enough.
 Convince the assembled classical mechanics that *your* solution is *the* correct one!
- (4) **Tennis ball- Basket ball Bounce:** Morin 5.23 plus demo
- (5) **More Damped, Driven, (Full Non-Linear) Pendulum:** *The* key characteristic of a chaotic system is “sensitivity to initial conditions”. Using Maple (or Mathematica) show that this pendulum is chaotic. Define the Liapunov exponent and use it to illustrate the transition to chaos.
- (6) **Rockets! Part I** As a prelude to Toy Week investigate a more realistic rocket problem. Using Maple (or Mathematica) solve for a rocket’s trajectory.
- (7) 4.23 E Deriving the amplitude correction for a pendulum
- (8) **Feigenbaum Numbers and Universality:**