Classical Mechanics (PHYS 350): Week 4

Fall 2013 v1.2

New Methods! Over the next couple of weeks we will reformulate mechanics so that all - ok, so that's a bit strong, "nearly all" - vestiges of Newtonian mechanics will vanish from the field of view! Morin does this in the "one fell swoop" approach in chapter 6. This suite of methods is an extremely powerful way to study dynamical systems. These methods form the cornerstone of much of modern theoretical physics.

Despite the prevalence of modern usage of these techniques began with the very old (1696) problem of finding the path which minimizes the time spent falling between two points. This is one of a class of questions which "launched a thousand methods." The quote is doubly appropriate because the problem provoked so many quarrels among mathematicians in the 17th century, that the cycloid became known as the "Helen of Geometers" (Boyer 1968, p. 389).[MathWorld] We will devote the next two weeks to study some of these - and will use the methods for the remainder of the semester.

The problem even found it's way into Moby Dick:

> It was in the left-hand try-pot of the Pequod, with the soapstone diligently circling round me, that I was first indirectly struck by the remarkable fact, that in geometry all bodies gliding along a cycloid, my soapstone, for example, will descend from any point in precisely the same time

(A try-pot is a large pot used to render blubber.)

The idea may seem odd at first: Find a function which solves the problem by calculating the extremum of an integral. But in some examples it makes sense. For instance in the brachistochrone problem in which we want to find the path which minimizes the time to fall between two points, we find the extrema of the total time.

Reading:

Chapter 6

You may wish to read about Fourier series. Taylor's *Classical Mechanics* sections 5.7-8 does the trick. Optional: There's a nice discussion in Chapter 2 of Hand and Finch, sections 2.1 and 2.2.

Problems: Due in the Tuesday-Wednesday class time window

(1) 5.3 P
(2) 5.9 P
(3) 5.16 P
(4) 5.26 P
(5) Fourier
   (a) Find the Fourier series for the periodic function if in one period
   $$f(t) = t \text{ on the interval } -\pi < t < \pi$$
   The function repeats every $T = 2\pi$.
   (b) If this function drives a damped oscillator, what is the Fourier series for the steady state solution $x(t)$? To answer this please use the notation we used in class, expressing the result in terms of $\omega, \beta, \omega_o, n$ and t.
(6) An astronaut wanders too far away from a space shuttle. The shuttle moves away at 3 m/s. The astronaut and the maneuvering unit have a mass of 100 kg, including a pressurized tank of mass 10 kg, with only 2 kg of gas in it. The gas escapes with velocity of 100 m/s.
   (a) Will the astronaut run out of gas before reaching the shuttle?
   (b) With what velocity will the astronaut have to hurl the empty tank to reach the shuttle?

Friday Class: Presentations