Classical Mechanics (PHYS 350): Guide 5

Fall 2013 v1.0

With this (coming) week, we move into the final part of the course, what I call "Applications and Detailed Study." We will apply the methods of Lagrangian (and Hamiltonian! more on this after break) mechanics to study a variety of physical systems including motion in *central potentials*, the subject of this week's new material. The classic example of central force motion is Newtonian gravity since the attractive force of gravity acts between two masses. We focus on this two-body system also known as the Kepler Problem. Later we go on to see what we can say for n-bodies.

The chapter incorporates a number of techniques to solve the two-body problem including reduced mass, angular momentum conservation, effective potentials, conic sections, and energy considerations. There is also some (new?) nomenclature to describe orbits such as perigee and apogee.

We will also work through further Lagrangian and Hamiltonian problems.

Reading:

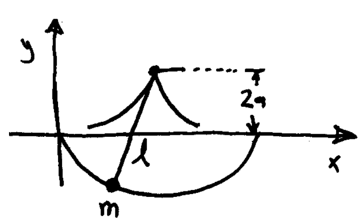
We have been playing in Morin's Chapter 6.
Taylor "Nonlinear mechanics and chaos" *Classical Mechanics* sections 12.1- 12.5 on the non-linear pendulum. It is on eReserves in the library. (The rest and the "Universality in Chaos" reading is also great, but optional.)
We will be moving into studying central potentials, also called the "Kepler problem" , discussed in Morin's Chapter 7.
For more on Hamiltonians read either 7.9-7.10 in Thornton and Marion or 5.1-5.5 in Hand and Finch (See the course information for more on these textbooks.)

Problems: Due Wednesday October 23

(1) (Our long postponed problem) As sailing and navigation developed the technology of keeping time failed to keep up with it, with disastrous consequences. Pendulum clocks don't work very well since the period of the clock depends on angular amplitude (as you recall from Alex and Kerkira's presentation); they are not isochronous. One way to make a better pendulum clock is to shorten the string as it swings. (Huygens (yup, same one as the waves) is given credit for working this out.) This "shortening" can be accomplished by installing curved metal "chops" in the shape of a cycloid. You can see them in many pendulum clocks today. These chops cause the pendulum mass to follow a cycloid with trajectory given by

$$x = a(\phi - \sin\phi), \text{ and } y = a(\cos\phi - 1)$$

with the length of the pendulum given by $\ell = 4a$. Compared to last week these are shifted and inverted so that the curves look like:



Show that this pendulum is exactly isochronous with natural angular frequency $\omega_o = \sqrt{g/\ell}$. You have local experts...

(2) More Fourier

(a) Find the Fourier series for the periodic function if in one period

$$f(t) = \begin{cases} -1 & \text{on the interval} \quad -\frac{\pi}{\omega} < t < 0 \\ +1 & \text{on the interval} \quad 0 < t < \frac{\pi}{\omega} \end{cases}$$

(b) If this function drives a damped oscillator, what is the Fourier series for the steady state solution $x(t)$? To answer this please use the notation we used in class, expressing the result in terms of $\omega, \omega_o, \beta, n$ and t.

(3) **Falling chain** Some time ago we derived an expression for the normal force on a scale as a chain fell. If I recall correctly, we found, for a chain of linear mass density λ and length ℓ, the normal force as a function of distance fallen y to be

$$N = \frac{\lambda g}{2(\ell - y)} \left(2\ell y - 7y^2 - \ell^2\right)$$

which runs off to infinity when the last bit of chain hits the scale ($y = \ell$). As you remember from the demo, this did not happen. Now after the falling chain presentations, revisit this problem and correct our solution. Clearly describe and defend any assumptions you make.

(4) **Non-linear pendulum sweetened with maple** Taylor 12.6, 12.8, and 12.14 (These are all concern the same damped, driven pendulum system.) We have code for this...

(5) A bead is constrained to move on a circular wire rotating at a constant angular speed ω about a vertical diameter. See Figure 6.15. Let's assume that there is no friction. Use lagrangian techniques for this one.
  (a) Find the equilibrium (or equilibria) angle(s) θ_o.
  (b) Calculate the frequency of small oscillations.
  (c) Find and interpret the critical angular velocity ω_c that divides the bead's motion into two types.

(6) 6.2 P

(7) 6.14 P

(8) 6.19 P *Except* do only the equal mass, equal length case.

(9) 6.25 E

Friday Class: Stay tuned...

- Brachistochrone 6.24
- Let's not forget this fun one! Bring in some soapy water and two circular rings of different diameter. Demonstrate Example 6-3 in Thornton and Marion. Explain in detail why a soap film would minimize surface area.