Classical Mechanics (PHYS 350): Guide 6

Fall 2013 v1.5

This past week we discussed small oscillations, constraints, variational problems, Noether's theorem, Hamiltonians, phase portrait, cyclic coordinates, momenta. This coming week we will work through the Kepler problem (planetary motion) by first working through what we know for all "central potentials" for which $U = U(r)$.

Reading:

We have been playing with Hamiltonians and the reading is in either Thornton and Marion 7.9-7.10 in Hand and Finch 5.1-5.5 (See the course information for more on these textbooks.) For central potential (or force) motion, also called the "Kepler problem" , see Morin's Chapter 7.

Problems: Due Wednesday October 30

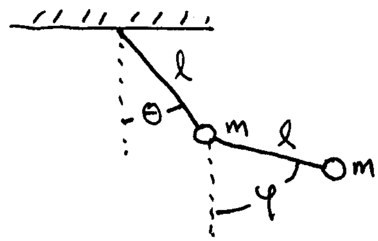
(1) 6.20 P shortest 'curve' in a plane
(2) 6.40 E Noether's theorem See 6.9 for ideas
(3) 6.13 P Small oscillations
(4) 6.43 E Small oscillations
(5) A spherical whale of radius ρ rolls without slipping on the lower half of a semi-circular half-pipe of radius R. Assume the whale is a uniform solid so that the moment of inertia is $I = \frac{2}{5}m\rho^2$. Use a constraint to solve this one.
  (a) Determine the Lagrangian, the constraint, and equation of motion for the one degree of freedom.
  (b) Find the frequency of small oscillations.
  We'll have a **Demo**!
(6) Revisiting a problem from last week: A bead is constrained to move on a circular wire rotating at a constant angular speed ω about a vertical diameter as in Figure 6.15. Let's assume that there is no friction. You found the Lagrangian,

$$L = \frac{1}{2}mR^2\dot{\theta}^2 + \frac{1}{2}mR^2\omega^2\sin^2\theta + mgR\cos\theta$$

  (a) Find the Hamiltonian. Rescale the Hamiltonian by dividing by $mR^2\omega_c^2$ with $\omega_c = \sqrt{g/R}$.
  (b) Using maple (or other program) plot the effective potential.
  (c) For an angular frequency ω larger than the critical angular velocity ω_c plot the phase portrait for several energies using maple (or other program) and a contour plot. We'll be plotting $p_\theta/mR^2\omega_c$ vs. θ.
(7) The Double Pendulum Returns! Starting with the Lagrangian in terms of coordinates (θ, ϕ), measured with respect to the vertical,

$$L = \frac{1}{2}m\ell^2\left(2\dot{\theta}^2 + \dot{\phi}^2 + 2\cos(\theta-\phi)\dot{\theta}\dot{\phi}\right) + mg\ell\,(2\cos\theta + \cos\phi)$$

  rescale by dividing by mgl and choose a new dimensionless, "natural" time coordinate $\tau = t\sqrt{g/l}$.



(a) For this system show the momenta are

$$p_\theta = 2\dot{\theta} + \cos(\theta - \phi)\dot{\phi} \text{ and } p_\phi = \dot{\phi} + \cos(\theta - \phi)\dot{\theta}.$$

(b) Find the Hamiltonian and Hamilton equations of motion.
Hint: The kinetic term of the Hamiltonian is

$$T = \left[\frac{p_\theta^2}{2} + p_\phi^2 - \cos(\theta - \phi) p_\theta p_\phi\right] \frac{1}{[2 - \cos^2(\theta - \phi)]}$$

(8) For any two functions, $g(q_i, p_j)$ and $h(q_i, p_j)$, of the N generalized coordinates and momenta, q_i, p_j the **Poisson brackets** are defined as

$$\{g, h\} = \sum_{k=1}^{N} \left(\frac{\partial g}{\partial q_k} \frac{\partial h}{\partial p_k} - \frac{\partial g}{\partial p_k} \frac{\partial h}{\partial q_k} \right).$$

If the Poisson bracket between two quantities vanishes then the quantities "commute". If the Poisson bracket of two quantities is unity then the quantities are "canonically conjugate." Verify that

(a) Assuming $g = g(q_i, p_j, t)$ show that

$$\frac{dg}{dt} = \{g, H\} + \frac{\partial g}{\partial t}$$

(b)

$$\dot{q}_i = \{q_i, H\}$$

(c)

$$\dot{p}_i = \{p_i, H\}$$

(d)

$$\{q_i, q_j\} = 0$$

(e)

$$\{q_i, p_j\} = \delta_{ij}$$

(f) Which quantities are canonically conjugate?
(g) Show that any quantity that commutes with the Hamiltonian is a constant of the motion.

Friday Class: Stay tuned...

- **Brachistochrone** 6.24 Work through the details of the solution I skimmed in class. Look up the history of this one. We have a **Demo**!
- **Rolling motion** Present your solution of 5 and show the demo.
- **Soap Bubbles** Bring in some soapy water and two circular rings of different diameter. Demonstrate Example 6-3 in Thornton and Marion. Explain in detail why a soap film would minimize surface area.