Classical Mechanics (PHYS 350): Guide 7

Fall 2013 v1.0

This week, we apply the methods of Lagrangian and Hamiltonian mechanics to motion in central potentials. We used number of techniques to solve this two-body problem including reduced mass, angular momentum conservation, effective potentials, conic sections, the magical u equation (or y in Morin - Spencer celebrates) and energy considerations. There is also some (new?) nomenclature to describe orbits such as perigee and apogee.

We will also work through further Lagrangian and Hamiltonian problems and return to the double pendulum to exercise our new tools to study this non-linear dynamical system.

Reading:

Morin's Chapter 7 is on central potentials. You may like a different take on this subject, try Fowles Chapter 6, Sections 5-12, Baierlein Chapter 5, or Hand and Finch Chapter 4 .

Problems: Due Wednesday November 6

(1) 7-1 P
(2) 7.4 P
(3) 7.5 P Using the magical u (or y) equation.
(4) Kepler's Third Law allows you to "mass" the sun. Determine the solar mass from the length of year and the mean radius of the Earth's orbit ($R = 1.49 \times 10^{11}$ m) neglecting the small eccentricity. This method can be used to find the mass of "suns" in distant planetary systems. See, for example, a press release from Jodrell Bank Observatory. (Sadly it seems that the companion star is not a black hole.)
(5) The comet Hyakutake was visible in the skies March-May 1996. If it has eccentricity .999846 and had a perihelion 0.230123 AU at 9:31 UT (universal time) on 1 May 1996, find when it is expected to return again. For more see more from NASA.
(6) The force between two particles, with reduced mass μ, is

$$F(r) = -\frac{k}{r^2} - \frac{\lambda}{r^3} \text{ with } k > 0,\ \lambda > 0$$

Show that the motion is a precessing ellipse. Consider the cases
(a) $\lambda < \ell^2/\mu$
(b) $\lambda = \ell^2/\mu$
(c) $\lambda > \ell^2/\mu$
(7) 7.21 E
(8) **Double Pendulum Numerical Set-up**: Next week we will numerically integrate the Hamilton equations of motion for the double pendulum and investigate chaos. Numerics is the way to go since these equations are non-linear and complicated as we found on Guide 6! This week we'll set this up and do some checks. The Hamiltonian may be expressed as

$$H = H(\theta, \varphi, p_\theta, p_\varphi) = \left[\frac{p_\theta^2}{2} + p_\varphi^2 - \cos(\theta-\varphi)p_\theta p_\varphi\right]\frac{1}{[2-\cos^2(\theta-\varphi)]} - 2\cos\theta - \cos\varphi$$

which is a function of 4 coordinates on phase space. Let $x = 2 - \cos^2(\theta - \varphi)$. The equations of motion are, for θ,

$$\dot{\theta} = \frac{1}{x}(p_\theta - p_\varphi \cos(\theta - \varphi))$$

and

$$\dot{p}_\theta = -\frac{1}{x^2}\left(p_\theta^2 + 2p_\varphi^2 - 2\cos(\theta-\varphi)p_\theta p_\varphi\right)\cos(\theta-\varphi)\sin(\theta-\varphi) + \frac{\cos(\theta-\varphi)p_\theta p_\varphi}{x} - 2\sin\theta$$

For φ,

$$\dot{\varphi} = \frac{1}{x}\left(2p_\varphi - p_\theta\cos(\theta-\varphi)\right)$$

and

$$\dot{p}_\varphi = -\frac{1}{2x^2}\left(p_\theta^2 + 2p_\varphi^2 - 2\cos(\theta-\varphi)p_\theta p_\varphi\right)\cos(\theta-\varphi)\sin(\theta-\varphi) + \frac{\sin(\theta-\varphi)p_\theta p_\varphi}{x} - \sin\varphi$$

Right. So to save your sanity let's have a template worksheet with these entered.

- Download the maple file.
- Set up the numerical integration for one double pendulum and test it for two initial sets of initial conditions: start at rest with $(\theta_o, \phi_o) = (\pi/20, \sqrt{2}\pi/20)$ and $(\theta_o, \phi_o) = (-\pi/20, \sqrt{2}\pi/20)$. Plot the angular positions $\theta(t)$ and $\varphi(t)$. What is the frequency of oscillation for these initial conditions? Why did I choose these initial conditions?
- Save your file so you can use it again.

Friday Class: optional presentations follow

- Let a central potential be expressed as $U(r) = k/r^n$. The *only* values for n for which the system has closed orbits are $n = 1$ (Keplerian) and $n = -2$ (Hookean) (!). This result is known as Bertrand's Theorem. Any other value of n yields orbits which precess (much like 8-13). Thus, precession of orbits is a sensitive test of the radial dependence of force between two masses. If the actual potential deviates from $1/r$ then we may very well see it in the precession of the orbit. For instance, the precession of the perihelion of Mercury is one of the classic tests of general relativity. Present either the proof of the theorem (in Goldstein Appendix A) or a numerical investigation (Hand and Finch offer some suggestions in Problem 29 on page 167.)
- Tell us about Hohmann transfers in Thornton and Marion section 8.8 and at http://liftoff.msfc.nasa.gov/academy/rocket_sci/satellites/hohmann.html and http://web.mit.edu/12.000/www/finalpresentation/traj/hohman.html.