Classical Mechanics (PHYS 350): Week 8

Fall 2013 v1.5

This week we study the dynamics of N particles, $N \geq 2$, (the N-body problem). This is a hard problem; the general analytic solution for $N > 2$ doesn't exist! However, there are useful techniques that allow us to accurately predict the dynamics of rigid, or fixed shape, and semi-floppy objects such as spheres rolling on the insides of cylinders, falling chains, and moving ropes.

Morin's work on this spans chapters 8 and 9. There's a good bit or review in this material but the KEY, KEY, KEY - sorry but I have a one year old in the house and repetition comes naturally - bits are the moment of inerta tensor (9.2 and 9.3), the Euler angles (9.7), and the Euler equations of motion (9.5). I am disappointed that Morin doesn't use the Lagrangian techniques, but we'll do these in class, and there is lots of other good stuff in the text.

Reading:

For reading on Rutherford scattering (expanding on Morin Problem 7.7) see **Quantum Physics** by Eisberg and Resnick pages 90-93. Alternately, a much more developed presentation is in Chapter 14 of Taylor.
Chapter 8 in Morin, which is a mix of new stuff and review
We'll be moving into Chapter 9 this week.

Problems:

Problems are due Friday November 15.

(1) **Black Hole Orbits** You take a long trip to a nearby black hole of mass M. Your spaceship orbits the very massive black hole at a safe distance. Mapping your orbit you find that your effective gravitational potential, up to an overall scale, is

$$\Phi_{eff}(r) = -\frac{GM}{r} + \frac{\ell^2}{2r^2} - \frac{GM\ell^2}{c^2r^3}$$

where c is the speed of light and ℓ is the (conserved) angular momentum. It is handy to plot this in terms of $\rho = r/M$ and $l = \ell/M$. For the moment let $G = c = 1$. In this case Φ_{eff} becomes

$$\Phi_{eff}(\rho) = -\frac{1}{\rho} + \frac{l^2}{2\rho^2} - \frac{l^2}{\rho^3}$$

  (a) Plot the effective potential for the black hole for $\ell/M = l = 4.3$ with $0 < \rho < 40$ and $-0.03 < U < 0.05$ in these units. Add the Newtonian effective potential to your plot. Compare the two potentials and describe how the orbits might differ in the two effective potentials.
  (b) Find the radii of circular orbits using the black hole effective potential.
  (c) Find the innermost sable circular orbit (ISCO) and the conditions for it to exist. Hint: Find the ℓ/M for which the orbit is "innermost".

(2) 7.18 E

(3) A mass has a trajectory given by $r = \alpha(1+\cos\phi)$ (α is a constant) under the action of a central potential. Draw a picture of the orbit and find the value of n if the potential varies as $1/r^n$.

(4) One Saturday morning you stare down at your blueberry pancake (15 cm in diameter) lying on your plate and count 6 wild blueberries, each 0.5 cm in diameter. Oddly instead of eating the pancake, you ask -
  (a) What is the cross section σ of a blueberry?
  (b) What is the target density n (number/area) of berries as seen from above?

(c) What is the probability that a fork, of width 3 cm, dropped at random into the pancake will hit a blueberry? (If you like, assume the fork is point-like. Clearly state this assumption is you use it.)

Note: Use the numbers only at the last possible moment.

N.B.: If all this seems less than familiar I recommend section 14.2 in Taylor. (If your future or past interests include scattering I heartily recommend the whole chapter.)

(5) 8.2 P

(6) 8.23 P (Optional, worth +1 point)

(7) 8.36 E

(8) 8.52 E Solve this one using the Lagrangian method

(9) Does the CM have to be in the object? Defend your answer with an argument or counter example.

(10) In a somewhat messy Halloween trick, a student fills a witch's hat with concrete, spilling some concrete on the rim of the hat so that it forms a disk. The final shape is a cone with height h and radius a attached to a disk of radius $2a$ and width w. Find the center of mass of the filled hat.

(11) **Double Pendulum Numerical Set-up**: This week we will numerically integrate the Hamilton equations of motion for the double pendulum and investigate chaos.
- Load your maple file from last week
- Play with a class of initial conditions specified by

  $$p_\varphi(0) = p_\theta(0) = 0 \text{ and } \varphi(0) = 0 \text{ and } \theta(0) \text{ in the range } 55 - 90 \text{ degrees}$$

  and find where in this range you suspect the system might become chaotic.
- For *two* double pendula, with initial conditions that differs from each other by 10^{-7} degrees, make a plot of $\ln|\delta\varphi|$ versus the dimensionless time τ. Use initial conditions $p_\varphi(0) = p_\theta(0) = 0$, $\varphi(0) = 0$, and $\theta(0) = 5, 25, 45, 65$ and 90 degrees. Find the sign of the Lyapunov exponent λ for each of these cases. Estimate the angle $\theta(0)$ at which the system become chaotic.
- Save your file so you can use it again.

"Friday" Class: Optional presentations

- **Rope Wrap** A rope of uniform linear mass density λ is wrapped around a wheel with moment of inertia I. The wheel rotated freely as the rope unwraps. One end of the rope can hang (and fall!) while the other is fixed to the wheel. Find the angular velocity as a function of angular displacement.
- Using the special apparatus and photogates, verify or correct the result to the last problem.