Classical Mechanics (PHYS 350): Guide 9

Fall 2013 v1.0

The new chapter in Morin examines the physics of motion in **non-inertial reference frames**. This requires two coordinate systems that I call inertial ("in") and non-inertial ("non-in") (also "fixed" and "body".) The theoretical development can be summed up rather simply: the study investigates position, velocity, acceleration and forces in these two systems. The basic idea is that we find additional acceleration due to the rotational motion of the frame itself. There are several "new" terms that appear in the acceleration in the non-inertial frame including Coriolis and centripetal accelerations. We will work with these terms in moving frames such as in the important case of the surface of the earth. The "new" accelerations can be multiplied by m and moved over to the "force" side of the equation. We call these new forces fictitious forces. On first encounter the application of these results can be very confusing - which is which?. The "Way Out" of Confusion is to be very clear about which reference frame you are referring to.

Reading:

Morin Chapters 9 and 10.

Problems:

Problems are due Friday December 6.

(1) 9.5 P
(2) A thin uniform rectangular plate (lamina) is of mass m and dimensions $2a$ by a. Choose a coordinate system such that the plate lies in the xy plane and has the origin at a corner. Find:
   (a) the moments and products of inertia
   (b) the moment of inertia about the diagonal through the origin
   (c) the angular momentum about the origin when the lamina is spinning with angular velocity ω around the diagonal, and
   (d) the kinetic energy for this case.
(3) As you slog your way back across campus through the snow you slip on the path. Your large self-portrait goes flying as you fall. In a moment of exceptional observational clarity, you notice as you fall that when the portrait initially rotates around one particular axis the rotation is unstable. Why does the portrait do this? You may answer this in the following way:
   (a) How are the moments of inertia I_1, I_2 and I_3 related?
   (b) Which is the axis referred to above?
   (c) Use the Euler equations to investigate the stability of rotation around the other two axes.
   (d) Summarize your results and use them to explain the portrait's behavior.
(4) A frisbee is thrown into the air with a definite wobble. If air friction exerts a frictional torque $-c\omega$ on the rotation of the frisbee, show that the component of ω in the direction of the symmetry axis decreases exponentially in time. Show also that the angle between the symmetry axis and the angular velocity vector ω decreases in time if the moment of inertia around the symmetry axis is larger than the other moment of inertia. (A frisbee is a symmetric top.) Thus, the amount of wobble steadily diminishes if there is air friction.
(5) 9.17 P
(6) 9.26 P
(7) 9.49 E (Optional, worth 1 pt)
(8) 10.9 P
(9) 10.16 E
(10) A ladybug crawls with constant speed in a circular path of radius b on a turntable rotating with uniform angular velocity ω. The circular path is concentric with the center of the turntable.

Of the mass of the bug is m and the coefficient of static friction is μ_s, at what speeds, relative to the turntable, can the bug crawl without slipping if it goes

(a) in the direction of rotation and
(b) opposite the direction of rotation?
(c) Which maximum speed is larger and why?

(11) Suppose that after Hurricane Irene the northward flowing Oriskany creek had a width of $b = 9.5$ m as it flowed under the College St. bridge. Suppose further that it moved at $v = 2.7$ m/s. The Coriolis effect causes one side of the river to be higher than the other side.

(a) On which side was the river deeper?
(b) Show that the difference in heights is

$$\frac{2bv\omega}{g}\sin\lambda$$

where v is the water speed, λ is the latitude, and ω is the earth's angular speed. Please neglect the earth's motion around the sun.
(c) What is the numerical value of this height difference?
(d) What would the situation be in the southern hemisphere for a similar southerly flowing river?

(12) 10.22 E