

The review problem set! This will take a few days to complete. You will also have questions. So the assignment is to email at least one question by Tuesday October 11, another by Friday October 14, and to turn in your problem set no later than 4 PM Thursday October 20. No late problem sets will be accepted. Each of the above emails is worth one problem, bringing the total to 9 points.

**Reading:**

We start Lagrangian methods Wednesday after fall break so please read T & M Chapter 7 Sections 7.1 - 7.6 *prior to Wednesday October 19.*

( For a more brief treatment, see Fowles Chapter 10. Hand and Finch have a presentation of these methods in Chapter 1.)

I will post the next guide after I get back. Enjoy!

**Problems:**

- (1) Growing up in Vermont, one of the great pleasures of the autumn was “whipping apples.” We launched apples using a flexible stick, typically a maple sapling. Some of our apple trees were on a hillside as pictured. We enjoyed seeing how far we could make them go. What angle  $\theta$  is optimal? If you want to make the apples go as far as possible, why would you whip apples on a hillside rather than on a level field?

- (2) Explain the mistake the editors made at the NYT in 1920.

*As a method of sending a missile to the higher, and even to the highest parts of the earth's atmospheric envelope, Professor Goddard's rocket is a practicable and therefore promising device. It is when one considers the multiple-charge rocket as a traveler to the moon that one begins to doubt ...for after the rocket quits our air and really starts on its journey, its flight would be neither accelerated nor maintained by the explosion of the charges it then might have left. Professor Goddard, with his “chair” in Clark College and countenancing of the Smithsonian Institution, does not know the relation of action to re-action, and of the need to have something better than a vacuum against which to react ...Of course he only seems to lack the knowledge ladled out daily in high schools. — New York Times Editorial, 1920*

- (3)  $N$  lemmings, each of mass  $m$  stand on a flat railway car. In the following they all follow each other off one end of the car, leaving at velocity  $u$  relative to the car. The car rolls in the opposite direction without friction.
- What is the final velocity if all lemmings panic and jump off at once?
  - What is the final velocity of the car if they line up and jump off one at a time? It is ok to leave your answer in terms of a series.
  - Which of the above cases yields the higher final velocity of the car? Can you find a simple physical explanation for your answer?
- (4) Show that light is a wave which travels at  $c$  by demonstrating that Maxwell's equations give the wave equation. Use Maxwell's equations in vacuum

$$\frac{\partial E^i}{\partial x^i} = 0 \quad (1)$$

$$\frac{\partial B^i}{\partial x^i} = 0 \quad (2)$$

$$\epsilon^{ijk} \frac{\partial}{\partial x^j} E^k = -\frac{\partial}{\partial t} B^i \quad (3)$$

$$\epsilon^{ijk} \frac{\partial}{\partial x^j} B^k = \mu_o \epsilon_o \frac{\partial}{\partial t} E^i \quad (4)$$

In SI units  $c = 1/\sqrt{\mu_o \epsilon_o}$ . You may find that the identity of 1-22 is useful.

- (5) In the Cavendish Experiment a torsion balance is used to determine Newton's constant,  $G$ . (You may have done this lab in freshman year.) A pair of lead balls each of mass  $m$  are suspended from a wire as shown. The experiment begins with no net gravitational attraction. At  $t = 0$  a pair of larger lead balls of mass  $M$  are rotated into close proximity of the smaller balls. The resulting angular position is sketched.
- Show that the equation of motion is given by

$$I\ddot{\theta} + b\dot{\theta} + \kappa\theta = \frac{RGMm}{r^2} \quad (5)$$

with  $b$  being the damping constant,  $R$  the radius of the small lead ball,  $r$  the distance between the centers of the two balls,  $\kappa$  is the torsional spring constant, and  $I$  is the moment of inertia. Use a the Reynolds number to explain why is the drag term should be linear.

- Assuming that the gravitational force is constant ( $r = \text{constant}$ ) find the solution to the differential equation. Be sure to use appropriate initial conditions. Use Maple to plot your solution.
- (Optional Bonus) Let's remove the incorrect assumption that the force is constant. Fine the dependence of  $r$  on  $\theta$  and use Maple to numerically evaluate the solution.

A few helpful numbers are  $R = 5.1$  cm,  $r = 4.5$  cm,  $M = 1.5$  kg. You'll have to play with your plot a bit to find  $\omega_o = \sqrt{\kappa/I}$  and  $2\beta = b/I$ .

- (6) One motivation for the search for “dark matter”, i.e. mass which does not emit light, comes from “rotation curves” of galaxies. These rotation curves are plots of velocity versus radius. (Velocities are measured using the Doppler shift.)
- (a) Model a galaxy by assuming that all the mass of a galaxy is concentrated at the galactic center. Calculate the velocity  $v$  of an orbiting star at a radius  $r$ . Assume that it is in a circular orbit.
  - (b) Sketch a rotation curve ( $v$  vs.  $r$ ) for your model.
  - (c) Google NGC 5194 (the “Whirlpool galaxy”) and examine the rotation curve for the galaxy. Based on your model explain why the curve is surprising. Notice that this curve levels off at about 8 kpc.
  - (d) Make a new mass distribution model for the Whirlpool galaxy with a spherical mass distribution

$$M = \rho(r) \frac{4}{3} \pi r^3.$$

Find the dependence of  $\rho$  on  $r$  such that you correctly model the rotation curve for  $r > 8$  kpc.

- (e) Looking at a photograph of the galaxy comment on the result. (1 pc or “parsec” short for “parallel arcsecond” is about  $3.08 \times 10^{16}$  meters.)
- (7) Sketch the phase space portrait for the given potential  $U(x)$ .