Classical Mechanics (PHYS 350): Toy Week

Fall 2013 v1.6

Extra Extra: Impress your friends! Confound your relatives!

We will apply our new methods of classical mechanics to a set of toys. We'll do our best to understand the Gryo-ring, rattleback, a "super bounce," a symmetric sleeping top, the Tippy Top, the Big Coin, and more!

Toys:

Choose one toy to investigate in detail. Develop an explanation of the physics involved in the dynamic behavior of the system. After you have done some initial work please come by so we can talk about your presentation.

Your presentation will be 10 minutes + 2 minutes for questions. To make this possible write up your results in the form of a handout. Plan on a brief description of the results and a demo. I can provide copies of your handout for everyone. These handouts may prove to be useful during the final exam.

There is a high degree of variability in these projects; some are quite hard, some less difficult. All are open-ended to one degree or another. Having done most (but not all!) of these projects I'm happy to give you an indication of the difficulty and time commitment involved in the projects.

The funny codes roughly indicate when your presentation will be - beginning, middle, or end of week.

Enjoy!

- **Bicycle Dynamics - MOW** (Connor) Ok, so what's the deal about keeping upright? See David E. H. Jones, "The stability of the bicycle", Physics Today, September 2006 and J. Fajans *Am. J. Phys.* 68 (2000) 654 and the refs in this article.
- **Foucault Pendulum - EOW** (Bennett) Install a Foucault pendulum in the North Atrium! In this project you will construct "version 4.0" of the pendulum. (earlier versions were by Josh Newman, Lucas, and Emi Birch working with Jim Schreve) The possibilities are many but the minimum is to construct a working pendulum and test whether the Science Center in fact rotates with Earth by comparing theory with experiment. (Perhaps it rotates about some other origin ...) You will likely work with Jim and the machinists. [References: see this html link and Scientific American magazine: pages 115-124 of the June 1958 issue and pages 136-139 of the February 1964 issue. Olsson *Amer. J. Phys.* **46** (1978) 1118 and *Amer. J. Phys.* **49** (1981) 531, Cane *Amer. J. Phys.* **63** (1995) 33; Schulz-Dubois, *Amer. J. Phys.* **38** (1970) 173.
- **Twisting Tennis Racket - BOW** (McKinley) It is not uncommon for idle tennis players to toss a racket into the air and catch the handle after one rotation. But what happens if you try to toss a tennis racket while attempting to rotate it around the axis in the plane of the net and perpendicular to the handle? It flips over! The racket typically makes a half rotation about the handle. Why? [Reference: Ashbaugh *at. al.* "The Twisting Tennis Racket" *Journal of Dynamics and Differential Equations* **3** (1991) 67. See also Morin 9.14 and 9.33]
- **Sleeping Top - BOW** (Will) When we set a top spinning in a stable configuration, the axis of rotation remains upright. This is the case of the sleeping top. After a little while, it slows down and "nods off," i.e. the axis tips over a bit. Investigate this motion for a symmetric top. Find the condition for stable motion. Show why this condition changes as the top spins down and make a plot of the effective potential for a variety of cases.
- **A Cylinder and a Sphere: The story and the data - BOW** (Alex G) As a warm-up show that for small oscillations of a uniform sphere rolling in a cylinder, the angular frequency

is

$$\omega = \sqrt{\frac{5g}{7(R-r)}},$$

or, is it? Experimentally test your result for ω using the Hamilton College demonstration. Now, assume that *both* the cylinder and the sphere move. Set up the Lagrangian and find the frequency of small oscillations.

- **Skipping Stones - EOW** (Malcolm) Folks who live by a body of water have a great opportunity for amusement - skipping stones. Flat stones are given spin and plenty of forward momentum. If aimed correctly, the stone bounces off the surface of the water a remarkable number of times... Model the physics of this activity [References: "Best Way to Skip a Stone" New York TImes Magazine Dec 12, 2004, link), L. Bocquet *Am. J. Phys* **71** (2003) 150, C. Clanet *et. al.* *Nature* **427** 29 (1 January 2004)].
- **Rockets! - EOW** (Vineeth) We have an altimeter (or can obtain a new) for model rockets. In this project we'll develop a detailed numerical model of rocket motion including thrust measurements, drag, and wind. You will make and check predictions of the altitude for the rocket flight, downloading the recorded flight path. I have a book on this.
- **Falling Slinky - MOW** (Kerkira) When a slinky is held from one end, stretched by its weight, and then released from rest, the bottom doesn't immediately accelerate since it doesn't (yet) "know" that it has been released. Analyze and demonstrate the curious motion as it collapses and falls. The model contains a combination of simple falling and wave behavior. [Reference: Cross and Wheatland, "Modeling a falling slinky" *Am. J. Phys.* **80** (2012) 1051.]
- **Frisbee - EOW** (Luke) Explain it! Be sure to think about aerodynamics, spin, the Bernoulli effect, and gyroscopic effects [References: Hubbard and Hummel, conference report, M. Schuurmans in *New Scientist* 28 July 1990, 37, http://biosport.ucdavis.edu/research-projects/frisbee-flight-simulation-and-throw-biomechanics/frisbee-flight-simulation-and-throw-biomechanics, Bloomfield, L.A., "The flight of the Frisbee", Sci. Am., April (1999), 132.]
- **Boomerang - MOW** (Jake) Explain how it works! [References: A. King, *Am. J. Phys* **43** (1975) 770; Pyle and Sidley; a book chapter on this.]
- **Tippy Top - EOW** (Sunrose) A Tippy Top has the odd behavior that, once spinning, it leans over and "hops" onto its stem. Explain this behavior qualitatively. Where does the energy come from to raise the center of mass? For a more extensive investigation, find the equations of motion (Euler's equations). Numerically integrate these equations to show the effect of the torque before the stem hits the floor. [References: R. Cross, "The rise and fall of spinning tops" *Am. J. Phys.* (2013) 280, R. Cohen *Am. J. Phys.* 45 (1977) 12; H. Soodak *Am. J. Phys.* **70** (2002) 815; several pages of text (see me for these).]
- **Big Coin - MOW** (Grace) A coin initially set spinning on its edge will begin to precess (wobble) as its spins decreases. With further loss of energy the coin will "lie down" with an increase in precession rate while the turning rate decreases. Explain this curious of a spinning coin in detail. Assume that the coin does not slip. [Hints: See Morin 9.24; show that the rolling condition is $\omega_3 = 0$; show that the precession of the line of the nodes increases as $1/\sqrt{\sin\theta}$; show that the rotation rate around the vertical decreases as $(\sin\theta)^{3/2}$.]
- **Gyro-Ring - MOW** (Josh) Explain how this toy works. On the packaging, "The Gyro-Ring is really just five little tops of unusual design strung on a metal ring. Do you remember how tops appear to start 'wobbling' as they slow down? The rotation of the spin axis is called 'precession' and causes the ring to press on one side of the hole in the spinning top. The contact point between the ring and the top can be thought of as a gear that changes the motion of the upward moving ring into the top's spinning motion." [Reference: Dallavis, "Rotational dynamics of spinning rings: the Gyro-Ring", (2008) Reed B.A Thesis]

- **Rattleback - MOW** (Alex H.) A very simple toy with very surprising behavior: rotated one way it merrily rotates until friction slows it to rest. Rotated the other, it protests by rocking back and forth and then rotating the other way! Explain this curious behavior in detail. [References: Jearl Walker *Scientific American* 241 (1979) page 172, *Rattlebacks and Tippe Tops; Roundabout: The Physics Of Rotation in the Everyday World*, 33-38, W. H.Freeman and Company, New York (1985); Crane, "How things work: The rattleback revisited." *The Physics Teacher* 29 (1991) 278-9; Hermann Bondi *Proc. R. Soc. Lond. A* **405** (1986) 265.]
- **Levitron - Demo Version - MOW** (Spencer) In one of the display cases it would be neat to install an example of magnetic levitation. The idea, as demonstrated in the commercial toy, is to spin-stablize a magnet. Explain the way the toy works, including a detailed discussion of the effective potential. (If you like write this up as a poster for the display case and work out techniques to ensure that the object will remain spinning for days.) [Simon *et. al.* AJP 65 (1997) 286; Berry, Proc. R. Soc. London, Ser. A 452, 12071220 (1996); Berry http://www.lauralee.com/physics.htm]
- **Pendulum Clock - BOW** Explore the role of the anchor escapement in the traditional pendulum clock. This is a way to introduce Green functions. [Reference: M. Denny *Eur. J. Phys.* **23** (2002) 449]
- **Falling Chain Revisited - BOW** You recall the interesting result for the tension of a falling chain. We discussed, roughly, how the finite size of the links "cuts off" the tension, yielding a large (as compared to the weight) but finite tension. Develop the model further to see find tension as a function of time and see if you can find the maximum tension in terms of the properties of the chain (radius of curvature and link size). Also see if you can develop the dynamics enough to find an expression for the speed of the end of the chain as the last link swings past the vertical position. We have data... [Reference: Tomaszewski and Pieranski, "Dynamics of ropes and chains: I, The fall of the folded chain" *New J. Phys.* **7** (2005) 45.]
- **Rope wrap: Is this another mistake?? - BOW** (Katie) A rope of uniform linear density λ, mass m is wrapped once around a wheel with radius R and moment of inertia I. One end of the hangs off the cylinder. The other end is fixed to the wheel. The wheel rotates freely around its axis as the rope unwraps. What is the angular velocity as a function of the angular displacement? Solve the problem using the Lagrangian method:
  (1) Use the obvious angular coordinate and find the Lagrangian and equation of motion.
  (2) Find the angular velocity as a function of angular displacement.
  (3) Using the rope wrap data on the website (or gathering your own data), and appropriate rescaling of your solution, check whether the solution in Thornton and Marion Example 9.5 (and your own) is correct.
  (4) Extra: Modify this analysis to account for the finite thickness of the chain.
- **Double Pendulum!** As you know Hamilton Physics has a pair of nearly-identical double *physical* pendula. It turns out that the equations of motion of the physical pendula are identical in form to the ones we solved for the massless rods and point masses. In Guide 8 we found the initial conditions when the pendula went chaotic. In the Physics Department version, however, the initial conditions are set by varying $\varphi(0)$ rather than $\theta(0)$. Re run your simulations to find the onset of chaos for the two-mass double pendulum. Check this with the built physical pendula. If you are able, try using the periods of the restricted pendula to test your assumptions I can show you these modes and give you the data. Demonstrate your findings. Explain any discrepancies you find.
- **"Eileen's Ice Tea" - BOW** You may have noticed, as Eileen did some years ago, that when you slide a pitcher of tea (or other beverage) to a friend across a table, the pitcher exhibits rather erratic behavior. Try it. Explain what you see and show your calculations.

Review:

I strongly recommend that you also use toy week for review. Concentrate on the methods of Lagrangian and Hamiltonian mechanics and the applications. These include nonlinear dynamics, rotational motion, physics in non-inertial reference frames, central potential motion, and oscillations. I recommend starting with problems, referring back to the text, class notes, or solutions when you need a reminder.