Quantum Theory (PHYS 450): Week 11

Spring 2008 v1.0

There are two or three major techniques we have left to uncover in Quantum Theory. The first is "perturbation theory". While we have powerful methods for deriving exact, analytical solutions for quantum systems, a surprising number of interesting problems are technically beyond us. Fortunately there are some approximation techniques called perturbation theory that significantly extend the range of problems we can address. Chapter 11 gets us started on time independent perturbation theory - a method computing energy levels and wavefunctions for systems in environments *slightly* different than those for which we already have solutions. For instance, finding energy levels for the hydrogen atom in a weak electric field (Stark effect). The conceptual structure of the method is relatively simple. Presented in 11.1, the idea is that the perturbed states are expressed as a power series. To learn how it all works it is best to work through this section in detail. The rest of the chapter is devoted to examples so we can see how the method works in practice.

Reading:

Townsend Chapter 10
Davies and Betts Chapter 8
Townsend Chapter 11, sections 11.1 - 11.4 (For an alternate treatment of the same material see Davies and Betts Chapter 10.)

Problems:

Problems are due at the beginning of seminar. Please make a copy of your solutions before you arrive.

(1) In light of the ground state of the hydrogen atom and your solution to 8.5, explain why the Bohr model is bogus. See the top of page 284.
(2) Cylindrical symmetry II 10.10. Your solution to 9.22 will be useful.
(3) 10.16 Refer to Chapter 7 for properties of the raising and lowering operators. You may recognize the solution to part (c) once you set up the problem. If so, simply right down the solution.
(4) Calculate the radius at which the radial probability density is at a maximum for the ground state of hydrogen. Compare this to the ground state expectation value $\langle r \rangle$ and comment these results in terms of measurements on the radial location of the electron.
(5) Consider the probability of measuring the electron to be within $\pi/8 \approx 23$ degrees of the $+z$ axis. Feel free to use Maple or Mathematica for the integration.
    (a) If the electron were equally likely to be found in any direction (a "uniform probability distribution") what would the probability be?
    (b) If the electron is in the $n = 2, \ell = 1, m = 0$ state what is the probability? Explain why your result is not surprising.
(6) (optional) "Bessels found in deuteron" The deuteron is a composite particle built of a bound state of a neutron and proton. The interaction potential may be modeled as

$$V(r) = -Ae^{-r/a}$$

with $A = 32$ MeV and a is about 2.2 fermi. The coordinate r is the proton-neutron separation. We'll consider bound states so that $E < 0$.
    (a) Write down the radial equation for $\chi(r) = rR(r)$ for the case $\ell = 0$. Perform the transformation to the new variable "zeta" ζ

$$\zeta = e^{-r/2a}$$

to obtain the differential equation in ζ. Verify that this is a Bessel equation. Determine the general solution.

(b) Impose the appropriate boundary conditions on R at large r and at $r = 0$. Describe how you find the energies.
(c) Find the ground state binding energy for the deuteron.
(d) Compare these energies with the "square well" model described in Townsend section 10.3.

(7) 11.1
(8) 11.2
(9) 11.7

Seminar Presentations:

Come to seminar with your presentation notes complete. Ask questions about your presentations before seminar.

- Dan T: Briefly explain how Alice and Bob can detect eavesdropping using the technique you mentioned last seminar.
- Jordan: Present the series solution to the radial equation for hydrogenic atoms starting with (10.15). Concentrate on the series solution (pgs 279-280) and finding the radial functions (pgs 283-284) from the recursion relation. Include a solution to problem 10.5. Show how we can find R_{30}. Feel free to draw from Townsend and Davies and Betts. Write up your presentation notes to distribute in seminar. Any algebra not done in the texts should go in these notes so that we all have a record of the complete solution. (From last week)
- Wex: In Chapter 1 of Davies and Betts "probability current density" is defined. Present this discussion (pages 12 - 14).
- Dan C: At the blackboard present the method of non-degenerate perturbation theory. Highlight the assumptions, notation and what it means, the key steps, and the formula we use. Fill in or skip steps as needed.
- Walter: Present a solution of Davies and Betts problem 10.2
- Emily: Review the perturbative and exact solutions to the harmonic oscillator with a dipole interaction. Present your solution to 11.2, showing that the two solutions agree to first order.
- Ruth: Present your solution to 11.7
- Nguyen: Tell us about the key idea of degenerate perturbation theory, how we can adjust the usual PT technique, and sketch the use of degenerate PT in the Stark effect.
- Dan T.: Present your solution to 6
- Mike: Find a solution to the following example final exam problem. Guide the seminar through to a group solution. A nonrelativistic particle of mass m moves in the potential energy

$$U(x, y, z) = \frac{1}{2}k\left(2x^2 + 4y^2 + z^2\right)$$

Let $\hat{H}$ be the Hamiltonian and $\omega = \sqrt{k/m}$.

(1) What is the energy spectrum?
(2) What are the four lowest energy levels?
(3) What are the degeneracies of these levels?