Quantum Theory (PHYS 450): Week 12

Spring 2008 v1.0

This week we launch into a detailed study of the fine structure of the hydrogen atom - the last part of chapter 11. The engine behind this is perturbation theory. We'll have a bunch of presentations on the calculations and effects. In the Chapter 12 reading we start on multi-electron atoms by considering the quantum effects of identical particles.

Reading:

Townsend Chapter 11, sections 5 - 9
Townsend Chapter 12, sections 1- 3

Problems:

Problems are due at the beginning of seminar. Please make a copy of your solutions before you arrive.

(1) 11.4
(2) 11.6
(3) 11.10
(4) 11.11
(5) Davies and Betts 10.4
(6) 12.2
(7) **Selection Rules** The dipole moment operator is defined as

$$\mathbf{d} = -e\mathbf{r}$$

where $\mathbf{r} = x\hat{i} + y\hat{j} + z\hat{k}$. Electric dipole transitions are produced by the perturbing Hamiltonian

$$\hat{H}_I = \mathbf{d} \cdot \mathbf{E}.$$

Dandy. So here's the question: Is there any limit on the possible transitions? And if so, what are they? Approach this using the dipole moment. The transitions involve the matrix element

$$\langle n'\ell'm' \mid \mathbf{r} \mid n\ell m\rangle$$

so the idea is to study $\Delta m = m' - m$ and $\Delta\ell = \ell' - \ell$. One way to answer these questions is to study $\langle n'\ell'm' \mid x \mid n\ell m\rangle$, $\langle n'\ell'm' \mid y \mid n\ell m\rangle$, etc. Useful relations include the cartesian coordinates in terms of spherical coordinates, Appendix B in Davies and Betts and

$$\sin\theta P_{m\,\ell}(\cos\theta) = \frac{1}{2\ell+1}\left[P_{m+1\,\ell+1}(\cos\theta) - P_{m+1\,\ell-1}(\cos\theta)\right].$$

$$\cos\theta P_{m\,\ell}(\cos\theta) = \frac{\ell-m-1}{2\ell+1}P_{m\,\ell+1}(\cos\theta) + \frac{\ell+m}{2\ell+1}P_{m\,\ell-1}(\cos\theta)$$

Seminar Presentations:

Come to seminar with your presentation notes complete. Ask questions about your presentations before seminar.

- Walter: The Hydrogen Atom Flow Chart! Present a summary of the quantization of (spinless, non-relativistic) hydrogenic atoms with a flow chart. The chart should include the major steps (and associated equations) in the quantization. Tell us where there are choices in the quantization method. (For instance, you can solve the radial equation with a series method or instead identify the associated Laguerre ODE.) Include key relations like the $u - R$ relation. Use Townsend's notation but look up at least one alternate presentation in the quantum library (or in your math methods notes...)

- Dan C: Present the tricks of finding the powers of $\hat{r}$ for hydrogenic atoms. See Das and Mellissinos page 325 and Appendix 11. See also the last problems of chapter 11. Be aware that Dan T and Wex will be consulting you...
- Wex: Special relativity ?!? Why is this relevant in the hydrogen atom? Explain. Discuss energy in the special relativity. Expand the expression for energy and thus derive the first relativistic correction to the kinetic energy. Evaluate the first order shift as done on pages 323-324. Consult with Dan C on the needed expectation values or work with him to present a solution to 11.16.
- Ruth: To set up the Spin-Orbit Coupling present Thomas precession. The reference in footnote 8 is decent. Your last line should be equation (11.93), with the correct factor.
- Dan T: Present the perturbation theory with spin-orbit coupling, starting with equation (11.93) and finishing with equation (11.111). Consult with Dan C on the needed expectation value of $1/|\hat{r}|^3$.
- Emily: Summarize the Darwin term and the full package of fine structure effects in section 11.7. Include energy level diagrams.
- Mike: Present the fascinating history of the Lamb shift, including the Shelter Island conference.
- Nguyen: Present the Zeeman effect, calculating the first order shift due to a magnetic field
- Jordan: Present the scoop on identical particles. Tell us about the effects in the helium atom. Guide us through the solution of 12.2.