Quantum Theory (PHYS 450): Week 13

Spring 2008 v1.5

In this penultimate week we finish Townsend's discussion of the periodic table and bonding and launch into (low energy) scattering. We'll spend a bit more time on the nature of symmetry and the wavefunction for identical particles. Unfortunately Townsend doesn't give a full introduction to this and the role of direct and exchange integrals. The main scattering technique we'll discuss is partial wave analysis. We will not be covering the Born approximation.

Next week we finish with a bit on the quantization on the electromagnetic field in Chapter 14

Reading:

Townsend sections 12.2 - 3. Skim the rest of Chapter 12. I prefer the presentation in Das and Mellisinos of the He atom, see pages 442-452
Townsend Chapter 13, sections 1, 4-6 *skip* sections 2 and 3. I like the presentation in Das and Mellisinos Chapter 11.

Problems:

Problems are due at the beginning of seminar. Please make a copy of your solutions before you arrive.

(1) 12.4 Hint: Use the angular frequency as the variational parameter.
(2) Review Problem: Construct the time-independent Schrödinger equation of a classical harmonic oscillator of mass m and restoring force constant k. Find the value of α for which

$$\langle x \mid \psi \rangle = A \exp\left(-\frac{1}{2}\alpha x^2\right)$$

is an eigenfunction of the equation. Calculate the corresponding eigenvalue in terms of the classical angular frequency $\omega = \sqrt{k/m}$.
(3) 13.9
(4) 13.12 Don't spend a lot of time trying to get his form for the tangent relation. Set up your BC's and then ask me if the algebraic path is not clear.
(5) Consider two indistinguishable particles ("1" and "2") of mass m in a one dimensional infinite square well which is centered on the origin and has width L. The particles interact via $V_{12}(x_1, x_2) = v\delta(x_1 - x_2)$ with $v > 0$. The Hamiltonian is

$$H = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_1^2} - \frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_2^2} + V(x_1) + +V(x_2) + V_{12}(x_1, x_2) \tag{1}$$

(a) Assume the particles are spin-0 bosons. Identifying V_{12} as the perturbation Hamiltonian, write expressions for the zeroth-order energies and wavefunctions for the ground state and a general excited state. Draw an energy level diagram, with quantum numbers and degeneracies, showing the unperturbed energy levels. Discuss qualitatively how you expect the perturbation to change the spectrum.
(b) Repeat part (a) for spin-1/2 fermions. Label the levels with singlet and triplet.
(c) Using the wavefunctions with correct symmetry, compute the first-order corrections to the ground states for parts (a) and (b).
(d) Calculation first order shifts for the first excited states for parts (a) and (b).

Seminar Presentations:

Come to seminar with your presentation notes complete. Ask questions about your presentations before seminar.

- Nguyen: Find your favorite description of the He atom energy levels (*not* Townsend). Discuss the direct ("J") and exchange ("K") integrals in He. Show how they are used in parts (c) and (d) of problem 5.
- Ruth: Help to clarify Townsend's presentation of basic scattering! Present 1D "scattering theory", i.e. the parts of 13.1 which concern 1D tunneling, generously reviewing the material from chapter 6. Show a video of a wave packet scattering off a square barrier.
- Dan C: Help to clarify Townsend's presentation of basic scattering! Starting with Wex's presentation on probability current, develop the 3D version of scattering in section 13.1 (Skip all the 1D stuff as Ruth will be discussing this.) Carefully define the cross sections and differential cross section, using pictures. Discuss their dimensions. Derive equations 13.21 (the solution to problem 13.2 should be very quick - see me if you don't see why) and 13.24.
- Walter: Present the partial wave expansion. Review the rather delicate argument that yields equation 13.70. Das and Mellisinos have a full description of the derivation of equation 13.60 in chapter 11.
- Wex: Present hard sphere scattering then discuss the total cross section and the funny factors of 2.
- Dan T: Discuss the partial wave expansion for a finite well.
- Emily: Present scattering off a delta-function shell potential, Das and Mellisinos pages 498-500. This is also your solution to 13.12.
- Mike: Present resonances, starting on page 389.
- Jordan: Guide us through the solution of the review problem, (2).