Quantum Theory (PHYS 450): Week 14

Spring 2008 v2.0

Here we are in the last week of the last year - and in chapter 14. Townsend's novel presentation is a hybrid approach somewhere in between non-relativistic quantum mechanics and quantum field theory. The key method is time-dependent perturbation theory. You'll see applications in the first problems and in the "Golden Rule". Also, we have several presentations from "odds and ends" topics that might be of interest.

Enjoy!

Reading:

Townsend Chapter 14

Guide to Reading: Read sections 14.1 and 14.6 like its core, which they are. (See also Das and Melissinos 5.2 of Griffiths Chapter 9.) Read the rest of the chapter as a tourist - take in the major sites. If you are going on in physics and/or this stuff interests you, read with more care, especially the sections after 14.6.

Problems:

Problems are due at the beginning of seminar. Please make a copy of your solutions before you arrive.

(1) 14.6
(2) 14.7
(3) 14.8
(4) 14.9 (a)
(5) **Van der Waals interaction** Griffiths problem 6.31
(6) Consider a spin-less particle in the 3D infinite square well with potential, $V(x, y, z) = 0$ if $0 < x < L, 0 < y < L, 0 < z < L$. Otherwise $V(x, y, z) = \infty$.
  (a) Write down the stationary states and the allowed energy levels. Hint: See page 190.
  (b) What is the energy of the ground state? Is it degenerate?
  (c) What is the energy of the first excited state? Is it degenerate?
  (d) Introducing the perturbation $H_1 = U_o$ if $0 < x < L/2, 0 < y < L/2$ and 0 otherwise, draw the box with the perburbation potential.
  (e) Find the first order correction to the ground state energy.
  (f) Find the first order correction to the first excited state. Use these eigenvalues to determine the associated states.

Seminar Presentations:

Come to seminar with your presentation notes complete. **Ask questions about your presentations before seminar.**

- Mike: Present resonances, starting on page 389. (from last week)
- Using Appendix E, convince us that 14.15 and 14.16 are correct.
- Ruth: Tell us about the Aharonov-Bohm Effect - the punch line on page 403 and the experimental verification. Note the simularity with problem 8.6
- Remind us of the Maxwell's equations in terms of potentials - experience the nostalgia of returning of Griffiths E&M text.
- Wex: Bring home the conclusion on the top of page 409 by setting up the problem (box, **A** expansion, ϵ notation, and all) and carrying through the calculation of one term on the board.
- Jordan: Summarize the use of quantum oscillators to quantize the electromagnetic field.
- Nguyen: Set up Townsend's treatment of the hydrogen atom in the electromagnetic field.
- Mike: Discuss the basics of time-dependent perturbation theory

- Emily: Tell us about the Schrödinger, Heisenberg, and Interaction pictures.
- Walter: Delight us with Fermi's Golden Rule and Spontaneous Emission.
- Spin-statistics: problem 14.5 and additional references, wikipedia is not bad here but do check out John Baez's Jazzy version http://math.ucr.edu/home/baez/spin(underscore)stat.html.
- Intermediate-Field Zeeman effect (see Griffiths page 281 and associated problems). Application of perturbation theory and Clebsch-Gordon coefficients
- Dan C.: Neutrons bound by a current carrying wire? Cool, long problem using the variational principle:
  (1) The Hamiltonian for the neutron is given by
  $$H = -\frac{\hbar^2}{2m}\boldsymbol{\nabla}^2 - \boldsymbol{\mu}\cdot\boldsymbol{B}$$
  where the nuclear magnetic moment is
  $$\boldsymbol{\mu} = g\mu_n\hat{\boldsymbol{S}}$$
  and the neutron g-factor is -3.83 ($< 0!$), the nuclear magneton, $\mu_n = e\hbar/2m = 5.05 \times 10^{-27}$ J/tesla, and $s = 1/2$. Set up your cylindrical coordinates (r, ϕ, z) so that the wire is along the z-axis.
    (a) Find the B-field of the wire.
    (b) Since the system is not spherically symmetric, $\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}$ does not commute with the Hamiltonian. However, $\boldsymbol{J}_z$ does. Also note that since $\boldsymbol{L}_z$ has integer eigenvalues and $\boldsymbol{S}_z$ is 1/2, then $\boldsymbol{J}_z$ is a half-integer.
    (c) Show that the angular wavefunctions
    $$\langle\phi \mid m\pm\rangle = \mathrm{e}^{\mathrm{i}m\phi} \mid +z\rangle \pm \mathrm{i}\mathrm{e}^{\mathrm{i}(m+1)\phi} \mid -z\rangle$$
    are eigenfunctions of $\boldsymbol{J}_z$
    (d) Find the expectation value of $\hat{\boldsymbol{S}}$ in the states $\mid m\pm\rangle$.
    (e) We have computed the spatial piece of $\hat{H}$ before, in problem 10.10 (Week 11). Find the total Hamiltonian. Spend a short moment seeing if this Hamilton yields a recognizable ODE with known solutions.
    (f) Using the trial wavefunction
    $$\langle r\,\phi \mid 0\,m\pm\rangle = \langle r \mid 0\rangle\langle\phi \mid m\pm\rangle$$
    with $\langle r \mid 0\rangle = r\mathrm{e}^{-\beta r}$ find the ground state energy of the bound neutron using the variational principle. The variational parameter is β. Note that you must first decide which of the states $\langle\phi \mid m\pm\rangle$ is the ground state!
    (g) Calculate the expectation value of the radius r for this bound state.
    (h) Hey, maybe this could be used to trap neutrinos! How might this work?
- Dan T.: Read section 2.1 "Do Photons Exist?" in Greenstein and Zajonc. Using the results of Lamb and Scully show that the photoelectric effect may be explained *without* photons. Derive equation 2.4.
- Read section 6.2 "Bohm's Nonlocal Hidden Variable Theory" in Greenstein and Zajonc and tell us what Bohm's theory is. This is a warmup for non-conventional quantum mechanics