Quantum Theory (PHYS 450): Week 3

Spring 2008 v1.0

This week we will finish up Chapter 2 in Townsend and jump into 3D in Chapter 3. in which we see three dimensional state spaces and angular momentum.

Enjoy!

Reading:

Townsend Chapter 2, review sections 2.7-2.8.
Townsend Chapter 3, sections 3.1 - 3.6

Notes on text:

- page 68: This note on the relations of angular momenta is extremely important. It is part of the "if it quacks like a duck it is a duck" methodology of quantum theory (and physics). That is if some operator satisfies the same algebra as angular momentum then it **is** angular momentum.

Problems:

Problems are due at the beginning of seminar. Please make a copy of your solutions before you arrive. I have included several quick problems at the beginning.

(1) What is the identity operator "1" in the polarization basis? I am asking for the expression analogous to equation 2.57.
(2) Are the projection operators P_+ and P_- Hermitian? Use the z-basis matrix representation to answer this.
(3) Is

$$\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

Hermitian?
(4) 2.11
(5) 2.15
(6) 2.17 Birefringence ("double refraction") occurs when light passes through certain types of material, such as calcite crystals, and depends on polarization. This effect can occur only if the structure of the material is anisotropic. If the material has a single optical axis birefringence can be formalized by assigning two different refractive indices to the material for different polarizations.
(7) 2.19 *unless you finished this for last seminar*
(8) 3.1
(9) 3.2 The eigenvector problem for $\hat{S}_n$
(10) 3.3 Pauli matrix identities
(11) 3.4 Pauli matrix identities
(12) 3.10
(13) We are used to expressing angular momentum as $\mathbf{L} = \mathbf{r} \times \mathbf{p}$. On the first page of Chapter 7 in Davies and Betts you can find the operator expressions for the angular momentum based on this expression. Find
    (a) $\hat{L}_x \hat{L}_y$. When you do this calculations notes that, for instance,

$$y\frac{\partial}{\partial z} z \frac{\partial}{\partial x} = y\frac{\partial}{\partial x} + yz\frac{\partial}{\partial x}\frac{\partial}{\partial z}$$

    due to the product rule.
    (b) $\hat{L}_y \hat{L}_x$

(c) Subtracting these results show that

$$\hat{L}_x\hat{L}_y - \hat{L}_x\hat{L}_y = [\hat{L}_x, \hat{L}_y] = i\hbar\hat{L}_z$$

Seminar Presentations:

Please stop by or email me if you have any questions on your presentation. Come to seminar with your presentation notes complete. If you have any questions I am happy to help out.

- Walter: Summarize the important properties of Dirac notation and matrix mechanics using the summaries in sections 1.6, 2.8, and 3.8. Could you update your "VECTOR BRAKET" notes to include operators (and operator properties) and photocopy them to distribute in seminar?
- Everyone: Lingering questions on chapter 2 – This is your moment(s) to fill in any gaps you have in the problem solutions or the text. I will present two solutions of 2.4
- Emily: Show us, say by using a book, the rotations do not commute. Show us that the mathematics of matrix multiplication replicates this behavior (pages 66-67).
- Dan C.: Picking up where Nguyen leaves off, convince everyone that equations $(3.14a) - (3.14c)$ hold - the commutation relations for $\hat{J}_i$.
- Mike: Present your solution to problem 13 - the commutation relations for angular momentum.
- Ruth: Show that $\hat{J}^2$ and $\hat{J}_z$ have simultaneous eigenstates.
- : Wex: Introduce raising and lowering operators and explain precisely why we call them by those names (pages 73-74)
- Nguyen: Derive the eigenvalues of $\hat{J}^2$ and $\hat{J}_z$. (pages 74-77)
- Dan T: Present a solution to 3.7
- Jordan: Derive the uncertainty relation of equation 3.74 and highlight equation (3.75)