Quantum Theory (PHYS 450): Week 4

Spring 2008 v1.0

Through the mathematical machinery built up from the simple Stern-Gerlach experiments we discover three surprising results: (1) The three components of angular momentum (including spin) can not be simultaneously specified or, in fact, exist. (2) From the algebra of angular momentum operators we can completely describe the possible states of angular momentum states. (3) Schrödinger's equation is simply the result of probability conservation and time translation. Third, Before a week ago I expect it would not have been possible to anticipate these results.

We are nearly done in setting up the language of quantum mechanics. Much of the physics to follow is unraveling what this language actually means for the observation of quantum systems. Townsend begins this in chapter 4 with the dynamics of two-level or two state systems.

I hope that I have re-adjusted the seminar presentation topics and the problems in a way that reflects our conversation on Thursday. I have (1) Assigned mostly chapter 3 problems and (2) selected presentations with material from outside the book (articles and other texts). Comments on the changes are welcome!

Enjoy!

Reading:

Townsend Chapter 3, sections 3.7 and 3.8
Townsend Chapter 4

Notes on text:

- page 78-9: The importance of this uncertainty result is hard to overstate. For fun you might list the minimal list of assumptions which gives this result.
- page 94: Shocking how Schrödinger's equation just falls out, no? What is "Schrödinger's equation" for rotations?
- pages 104-106: Key Stuff: Work through this carefully
- pages 108-113: This analysis applies to any two-state system. It may seem odd but there are many systems which are effectively two-state.
- page 115: Note that Δt is weird. We'll have a presentation on this next week.

Problems:

Problems are due at the beginning of seminar. Please make a copy of your solutions before you arrive.

(1) 3.5 Finding $\hat{S}_n$ another way
(2) 3.6 Finding eigenvalues for $\hat{J}_-$
(3) 3.8
(4) 3.9 only complete the first part, not the second part on $\hat{S}_x$
(5) 3.14 Moving into spin-1 land...
(6) 3.17
(7) 3.20
(8) 3.21
(9) 3.22
(10) 4.1
(11) 4.4

Seminar Presentations:

Come to seminar with your presentation notes complete. If you have any doubt about your presentations, please ask questions before seminar.

- Dan T: Present a solution to 3.7 (from last week)

- Jordan: Derive the uncertainty relation of equation 3.74 and highlight equation (3.75). Please prepare a handout with all the algebraic steps. This is a fundamental derivation. (from last week)
- Nguyen: Review the spectral theorem (and specifically the spectral decomposition) of linear algebra. See if you can show why we label states with the eigenvalues of commuting, Hermitian operators.
- Walter: From continuum time and conservation of probability, derive Schrödinger's equation (pages 93-94). Show what would happen if you assumed that there was a "smallest unit of time" δt". What would replace the Schrödinger equation? Speculate on how this might be observed.
- Mike: Discuss a spin 1/2 particle in a magnetic field and how to use this system to measure g. Look up the 1957 article (it is available online through the library website). (pages 97 -101)
- Dan C: Recall the overall phases in equations 2.43a and 2.43b. Review the argument in section 4.3 to show that these phases are observable. Read the PRL cited in footnote 6. In seminar carefully explain the experiment and the interpretation. Include the following problem. Suppose the overall phase shift was $-i$, what would be the new ΔB be?
- Ruth: Discuss the quantum mechanics of NMR including a discussion of pages 104-108 and Das and Melissinos pages 204-211. Relate this description to the previous descriptions you have used. (Wex will derive Rabi's formula week.)
- Wex: Present your solution to 4.9. Start with equation 4.41. In seminar highlight the key steps and bring photocopies of the derivation.
- Emily: Present a solution to 4.2 and find the stationary states of the electronic states of a 1D system with

$$\hat{H} = \sum_{n=1}^{3} E_o \mid n\rangle\langle n \mid + \sum_{n=1}^{3} a \left(\mid n\rangle\langle n+1 \mid + \mid n+1\rangle\langle n \mid\right)$$

  Assume the states $\mid n\rangle$ are orthonormal; E_o and a are real parameters; and the states are periodic so that $\mid 3+j\rangle = \mid j\rangle$. You can find the stationary states by first finding the eigenvalues then eigenvectors.
- Seth (if time): Doing angular momentum with spin networks! If there is time I'll introduce spin networks and show how we can repeat angular momentum calculations using diagrams. See *Am. J. Phys.* **67** (1999) 972-980 for details.