Quantum Theory (PHYS 450): Week 9

Spring 2008 v1.0

We have largely confined our study of quantum mechanics to spin and 1 spatial dimension. Now it is time to generalize to three dimensions! This is what Townsend does in Chapter 9. As the saying goes, the generalization is "straightforward". However there are some unexpected aspects to this new material. First, Townsend emphasizes symmetries of the problem. At least locally on atomic scales, all directions look the same - space is "isotropic" - and no one coordinate location is special - space is "homeogeneous". These give rise to two symmetries, rotation and translation. Second, he introduces the key concept of "a complete commuting set of obervables" which tell us how we should label states. There are also some expected aspects of this generalization. For instance the reduced mass and relative coordinates from classical mechanics are (re-)introduced.

Enjoy!

Reading:

Townsend Chapter 8
Townsend Chapter 9

Problems:

Problems are due at the beginning of seminar. Please make a copy of your solutions before you arrive.

(1) 8.4 A quick estimate of the action for a neutron
(2) 8.5 A calculation of the action for electrons; assume straight line path
(3) 8.6 With a scalar potential V the Lagrangian gains the term $-qV$
(4) Using the WKB method find the approximate solution (wavefunctions and energy levels) for the "bouncing ball" problem that Jordon presented in Week 7. Note that you will find the energy spectrum for *large* energy.
(5) 9.1
(6) 9.2
(7) 9.3
(8) 9.4
(9) 9.9 The Hamiltonian operator is $\hat{H} = \hat{L}^2/(2I)$ with the moment of inertia $I = \mu r_o$. The idea is to find r_o.

Seminar Presentations:

Come to seminar with your presentation notes complete. Ask questions about your presentations before seminar. I'll be checking my email regularly during the first week of break.

- Dan T: Finishing guiding us through the discussion of the "which paths count" in section 8.6 (from before break)
- Dan C.: Digest the argument on pages 229-231 for us. Refer to Feynman's book QED, especially chapters 2 and 3 (from before break)
- Ruth: Explain how we take the general two body problem (see Hamiltonian operator 9.26) with no external potentials and reduce it to a *one* body problem (see Hamiltonian operator 9.57). Review briefly what is done in classical physics then emphasize the new aspects of this problem in quantum mechanics.
- Nguyen: Give us a qualitative overview of spherical harmonics. What are they? Show us some using a computer animation program such as "Atom in a Box" (http://daugerresearch.com/orbitals/index.shtml). (If the program shows hydrogen wave functions, just display the wave functions for a single value of n.) Show some superpositions of Y_{lm}'s.

- Jordan: Present 9.17, the series solution for the angular wave functions. Consult a Mathematical Physics text such as Boas or Weber & Arfken (See problem 8.5.5).
- Mike: Present the definition of a "complete commuting set of obervables". Start with Townsend's example of a set (introduced at the bottom of page 253) and lead a discussion of the general description what this means. See, for instance, Dirac's *Principles of Quantum Mechanics* page 57.
- Wex: So (9.92b) looks funny, doesn't it? Derive this form of the the momentum operator starting from $\langle \mathbf{r} \mid \hat{\mathbf{p}}^2 \mid \psi \rangle = -i\hbar \nabla^2 \langle \mathbf{r} \mid \psi \rangle$ in spherical coordinates.
- Walter: Solve 6.13 for the square well presented in problem 6.10. Repeat part (a) for a simple harmonic oscillator ground state that undergoes a sudden transition from the potential $V(x) = \frac{1}{2}kx^2$ to $V(x) = \frac{1}{2}(k/1600)x^2$ so that the angular frequency is reduced by a factor of 40. Present the discussion in light of Bruce Allen's review gr-qc/9604033 section 5, particularly pages 26 and 27 and thus explain how the problem relates to particle production in the early universe. The paper is available on the ArXiv site. I have link on my home page and on the 450 course website.
- Emily: Present the rigid rotator model and the energy spectrum (in section 9.7). Present your solution to 9.9.
- Dan T: Tell us about quantum cryptography and quantum teleportation. For a reference start with *The Quantum Challenge* by Greenstein and Zajonc.