Electrodynamics (PHYS 480) First Guide

Fall 2007 v1.5

So you know the theory already:

$$\begin{aligned}
\nabla \cdot \mathbf{E} &= \frac{1}{\epsilon_0}\rho \\
\nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\
\nabla \cdot \mathbf{B} &= 0 \\
\nabla \times \mathbf{B} &= \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}
\end{aligned} \tag{1}$$

For the rest of the semester we unravel many consequences of the theory.

To begin we'll jog our memory of all the mathematical symbols (er, well, vector calculus) used in Maxwell's equations. There is vector algebra and calculus, a bit on curvilinear coordinates, the Dirac delta function, and even a section on vectors fields in general.

This first week is unusual. Normally I will distribute the problem set the week before. Since you only have a couple days for these, the required list (denoted by $*$) is short.

Problems of note:

- * 1.2 Start with playing around with a few examples. There will be more on this later in the chapter.
- 1.4 Cross product practice
- 1.5 If you like try using the antisymmetric ϵ_{ijk} (See notes below).
- * 1.7 Working with script r in a simple context.
- 1.8 What is the general property which preserves length? If you know some group theory, what is this called?
- 1.10 Understand what a pseudo-X is.
- 1.12 Working with the gradient and a “hill”.
- * 1.13 Used later (that's what the dot means). Include on your formula sheet.
- 1.14 Well, he says that vectors are hose things that transform correctly under a change in coordinates so we had better check that the gradient is a vector!
- * 1.16 We'll return to this. The plot will thicken...
- 1.19
- 1.20 Try (i) and (iv)
- 1.37 Playing with spherical coordinates
- 1.38 ...it thickens
- * 1.43 (a), (d) and 1.44 (a), (c)
- * 1.47 (a)
- 1.60 More integral results. At least read this one through carefully.
- Example 1.15 This is a derivation to know and a formula to put on your sheet. Note how Griffiths shows the formula. He uses a *change of variables.* He could do nothing with only the left hand side (LHS) of Eq. (1.94). This is an example of what I mean by these objects “live happily under integrals.”
- Using the appropriate “fundamental theorem” of calculus, convert all of the Maxwell equations to integral form. You will need to “invent” four additional quantities. They will be familiar from previous studies... (Hint: Look at the LHS, integrate over what you can (surface or

volume), and use the appropriate theorem to dress the LHS in new duds. The RHS must be integrated as well, of course.)

- Combine the time derivative of the first line and the divergence of the fourth line of Eq. (1) to find an equation which relates charge density ρ to the current density. What is this equation called?

Notes on text:

- Read the "Advertisement" first. It is critical to have an understanding of the placement of this field of physics in the larger picture. We will return to this at the end of the course as well. Any idea why Griffiths leaves out "the other dimension" or realm of mechanics, general relativity? Another question to ponder is this: If electromagentic forces are the "dominant ones in everyday life" then why does gravity determine the large scale structure of the universe (including the existance of a space for electromagnetic forces to act!)? Griffiths does not mention the "range of validity" of electrodynamics, i.e. when we can reasonably expect the theory to give sensible answers. The best theories are those which tell us in what regimes they are valid. Any ideas on deciding when the classical electrodynamical description is adequate?[1]
- Chapter 1: page 3 Perhaps you are familiar which the antisymmetric or Levi-Chivita symbol, ϵ_{abc}. The "epsilon" is completely antisymmetric in pairs of indices so that, e.g. $\epsilon_{123} = 1$ while $\epsilon_{213} = -1$. It provides another way of working with vector products. For instance, a cross product for a vector $\mathbf{A}$ (or, equivalently A^a) is

  $$(\mathbf{A} \times \mathbf{B})^a = \epsilon^a_{bc} A^b B^c$$

  (The position of the indices is only critical when working in more general spaces; vectors have indices "upstairs.") The triple cross product is written as

  $$(\mathbf{A} \times (\mathbf{B} \times \mathbf{C}))^a = \epsilon^a_{bc} A^b \epsilon^c_{df} B^d C^f.$$

  How would one write the triple product $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C})$? For more complicated products the identity

  $$\epsilon^{abc} \epsilon_{cdf} = \delta^a_d \delta^b_f - \delta^b_d \delta^a_f$$

  is useful.
- page 8 A way of remembering the "BAC-CAB" rule for $\mathbf{A} \times (\mathbf{B} \times \mathbf{C})$: The first term of the expansion contains the middle vector with the scalar coefficient being the dot product of the other two vectors. The second term carries a negative sign and contains the "last vector" (the other one inside the parentheses). The scalar coefficient is the dot product of the remaining two vectors.

  If you have been sleeping through the first sections wake up a bit for the bit on Griffiths script $\mathbf{r}$ conventions. He uses them extensively.
- page 10 As good as this book is there are some mild faults. In section 1.1.5 Griffiths makes the point, "If it transforms like a vector, it is a vector." The (subtle) problem with this approach is that vectors exists independently of any components or coordinate systems. (Griffiths does point this out in the footnote on page 39.) This way of defining vectors is the "old way." A nicer way to define vectors is to base them on curves. A curve $\gamma(t)$ is a function which gives a point in the space if it is fed a number (the parameter value of t). At a point on the curve, a variation in the parameter t, d/dt, yields a magnitude and a direction - a vector! This variation also, of course, is linear and satisfies the product rule. So we define a vector at this point as the derivative. Why is this "nice"? First, there is no mention of coordinates. Second, it does not use displacements over finite separations (bad because in general spaces, "a vector here is

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[1] This is not easy! In a similar situation, Einstein and Grossmann missed one condition for determining the classical limit of general relativity and this cost Einstein five years of time to straighten out!

not the same as the vector over there"). Third, this definition emphasizes that the derivatives generate a "motion" along a curve as one would expect for a tangent vector. By the way we can still keep our old notions of vectors (at least at a point) by choosing the curves to be the usual coordinate frame.

- page 13 - the "heart" of the chapter. This is what we need to understand the Maxwell equations. Gain fluency by running through problems 1.11, 1.12, 1.15, and 1.18. Griffiths has great "hands on" descriptions of divergence and curl. Try giving three dimensional examples. There is an explosion of product rules in section 1.2.6 and 1.2.7. Take these slowly. Understand each one (what do the symbols mean? where does it come from? does it seem reasonable? where can we use it? does the order of factors matter?)
- page 24 Section 1.3 is the second major subject of any calculus – integrals. Griffiths builds them up studying integrals in 1,2, and 3 dimensions before turning to their relations (Green's, Stokes', and divergence theorems). He labors through several examples. If this material is at all rusty for you, these examples are a gold mine. Use them! (Work the problem without looking at the solution. When you run into the answer or a problem, greet it, and return to the example to check your solution with Griffiths'.) Check your understanding with a subset of 1.18, 1.29, and 1.30. The theorems have an overall structure: They relate integrals in $n + 1$ dimensions to ones in n dimensions, e.g. 1 and 0 (which is the fundamental theorem of calculus). By the way, the idea of equating the integration over a volume (in any dimension) to its boundary is surprising and deep.
- page 38: Carefully work through all this (up to Eq. (1.70)) so that you can recreate it at a moment's notice. We will have to several times this semester.
- page 46: The Dirac delta "function" $\delta(x)$ is not a function but a *distribution.* A raw, mathematically imprecise, definition is "distributions can only be evaluated inside integrals." If you end a calculation with the RHS of Eq. (1.88), for example, this is bad news. The result might as well be ∞. Nevertheless, the formulae Griffiths sprinkles throughout this chapter are useful. Do a couple of the integrals on page 49. Deltas make integration easy!
- page 50: Important material about the 3D Dirac delta that we use over and over again.