

So you know the theory already:

$$\begin{aligned}
 \nabla \cdot \mathbf{E} &= \frac{1}{\epsilon_0} \rho \\
 \nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\
 \nabla \cdot \mathbf{B} &= 0 \\
 \nabla \times \mathbf{B} &= \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}
 \end{aligned} \tag{1}$$

For the rest of the semester we unravel many consequences of the theory.

In class we have jogged our memories of the vector calculus used in Maxwell's equations. Chapter 1 has vector algebra and calculus, a bit on curvilinear coordinates, the Dirac delta function, and even a section on vectors fields in general. I have also added an introduction to index notation for vectors, including  $\epsilon_{ijk}$ .

Chapter 2 is the first of what will be five (!) chapters on the kinematics of electromagnetism. This study of electro- and magneto- statics will comprise about half the material of the seminar. And we start with the most familiar electric field. "The Problem" that we are faced with (or, rather, electrodynamics is faced with) is stated in the first paragraph: Given a set of static or uniformly moving charges, find the electromagnetic fields. Many of the techniques introduced in this and later chapters help us with this problem. We will make many neat connections with fun things you may have bumped into in the past, characters such as Bessel and Legendre functions.

Much of the initial material is review. Keep me posted on the best pace for the course. In any case I recommend pushing the reading through Chapter 2 over the weekend.

Some of the comments may seem dated, since we have already talked about them. However, please at least skim them.

### Problems: (Due on Friday September 12)

- (1) 1.10 Understand what a pseudo-X is.
- (2) 1.13 Used later (that's what the dot means). Include on your formula sheet.
- (3) 1.22(a) and (b) Use index notation.
- (4) 1.38 Playing with unit vectors in spherical coordinates
- (5) 1.43(a) Play with derivatives of a vector in cylindrical coordinates
- (6) 1.45 Working with  $\delta(\cdot)$
- (7) 1.47 These are all Dirac  $\delta$ -functions.
- (8) Using the appropriate "fundamental theorem" of calculus, convert all of the Maxwell equations to integral form. You will need to "invent" four additional quantities. They will be familiar from previous studies... (Hint: Look at the LHS, integrate over what you can (surface or volume), and use the appropriate theorem to dress the LHS in new duds. The RHS must be integrated as well, of course.)
- (9) 2.5 The  $\vec{E}$  field of a loop of charge
- (10) 2.15 Gauss' law in spherical symmetry, with a varying density
- (11) 2.16 Gauss' law in cylindrical symmetry with spice

### Notes on text:

- Read the "Advertisement" first. It is critical to have an understanding of the placement of this field of physics in the larger picture. We will return to this at the end of the course as well. Any

idea why Griffiths leaves out “the other dimension” or realm of mechanics, general relativity? Another question to ponder is this: If electromagnetic forces are the “dominant ones in everyday life” then why does gravity determine the large scale structure of the universe (including the existence of a space for electromagnetic forces to act!)? Griffiths does not mention the “range of validity” of electrodynamics, i.e. when we can reasonably expect the theory to give sensible answers. The best theories are those which tell us in what regimes they are valid. Any ideas on deciding when the classical electrodynamical description is adequate?<sup>1</sup>

- Chapter 1: page 3 Perhaps you are familiar with the antisymmetric or Levi-Civita symbol,  $\epsilon_{ijk}$ . The “epsilon” is completely antisymmetric in pairs of indices so that, e.g.  $\epsilon_{123} = 1$  while  $\epsilon_{213} = -1$ . It provides another way of working with vector products. For instance, a cross product for a vector  $\mathbf{A}$  (or, equivalently  $A^j$ ) is<sup>2</sup>

$$(\mathbf{A} \times \mathbf{B})^i = \epsilon_{jk}^i A^j B^k$$

(The position of the indices is only critical when working in more general spaces; vectors have indices “upstairs.”) The triple cross product is written as

$$(\mathbf{A} \times (\mathbf{B} \times \mathbf{C}))^a = \epsilon_{bc}^a A^b \epsilon_{df}^c B^d C^f.$$

How would one write the triple product  $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C})$ ? For more complicated products the identity

$$\epsilon^{abc} \epsilon_{cdf} = \delta_d^a \delta_f^b - \delta_d^b \delta_f^a$$

is useful.

- page 8 A way of remembering the “BAC-CAB” rule for  $\mathbf{A} \times (\mathbf{B} \times \mathbf{C})$ : The first term of the expansion contains the middle vector with the scalar coefficient being the dot product of the other two vectors. The second term carries a negative sign and contains the “last vector” (the other one inside the parentheses). The scalar coefficient is the dot product of the remaining two vectors.

If you have been sleeping through the first sections wake up a bit for the bit on Griffiths script  $\mathcal{Z}$  conventions. He uses them extensively.

- page 10 As good as this book is there are some mild faults. In section 1.1.5 Griffiths makes the point, “If it transforms like a vector, it is a vector.” The (subtle) problem with this approach is that vectors exist independently of any components or coordinate systems. (Griffiths does point this out in the footnote on page 39.) This way of defining vectors is the “old way.” A more general way to define vectors is to base them on curves. A curve  $\gamma(t)$  is a function which gives a point in the space if it is fed a number (the parameter value of  $t$ ). At a point on the curve, a variation in the parameter  $t$ ,  $d/dt$ , yields a magnitude and a direction - a vector! This variation also, of course, is linear and satisfies the product rule. So we define a vector at this point as the derivative. Why is this “nice”? First, there is no mention of coordinates. Second, it does not use displacements over finite separations (bad because in general spaces, “a vector here is not the same as the vector over there”). Third, this definition emphasizes that the derivatives generate a “motion” along a curve as one would expect for a tangent vector. By the way we can still keep our old notions of vectors (at least at a point) by choosing the curves to be the usual coordinate frame.
- page 13 - The “heart” of the chapter. This is what we need to understand the Maxwell equations. Gain fluency by running through problems 1.11, 1.12, 1.15, and 1.18. Griffiths has great “hands on” descriptions of divergence and curl. Try giving three dimensional examples.

<sup>1</sup>This is not easy! In a similar situation, Einstein and Grossmann missed one condition for determining the classical limit of general relativity and this cost Einstein five years to straighten out!

<sup>2</sup>Beware that my  $ijk$  index notation is not the same as Griffiths’ in this chapter. He uses these as in  $\mathbf{v}_a$  is “another”  $\mathbf{v}$  vector than  $\mathbf{v}_b$ , rather than my use which is different *components*. Unfortunately his notation is not upwards compatible.

There is an explosion of product rules in section 1.2.6 and 1.2.7. Take these slowly. Understand each one (what do the symbols mean? where does it come from? does it seem reasonable? where can we use it? does the order of factors matter?)

- page 21 Product rules galore! With the the  $\epsilon$  notation you can re-derive these at a drop of a hat.
- page 24 Section 1.3 is the second major subject of any calculus – integrals. Griffiths builds them up studying integrals in 1,2, and 3 dimensions before turning to their relations (Green’s, Stokes’, and divergence theorems). He labors through several examples. If this material is at all rusty for you, these examples are a gold mine. Use them! (Work the problem without looking at the solution. When you run into the answer or a problem, greet it, and return to the example to check your solution with Griffiths’.) Check your understanding with a subset of 1.18, 1.29, and 1.30. The theorems have an overall structure: They relate integrals in  $n + 1$  dimensions to ones in  $n$  dimensions, e.g. 1 and 0 (which is the fundamental theorem of calculus). By the way, the idea of equating the integration over a volume (in any dimension) to its boundary is surprising and deep.
- page 38: Carefully work through all this (up to Eq. (1.69)) so that you can recreate it at a moment’s notice. We will have to several times this semester.
- page 45: The Dirac delta “function”  $\delta(x)$  is not a function but a *distribution*. A raw, mathematically imprecise, definition is “distributions can only be evaluated inside integrals.” If you end a calculation with the RHS of Eq. (1.88), for example, this is bad news. The result might as well be  $\infty$ . Nevertheless, the formulae Griffiths sprinkles throughout this chapter are useful. Do a couple of the integrals on page 49. Deltas make integration easy!
- page 50: Important material about the 3D Dirac delta that we use over and over again.
- page 59 Griffiths very nicely states the goal: Given the sources, electrodynamics determines the fields and forces.
- page 67 If (1) “Field lines begin on positive charges and end on negative ones” and (2) Positive and negative charges “occur in exactly equal amounts, to fantastic precision, . . .” then how is it possible to have field lines extending to infinity?
- pages 68-9 We did this derivation the “other way”, from Maxwell’s equations. Note that the physics you have to invoke is different depending on the direction of your derivation, e.g. the total charge was already defined in Griffiths derivation while the density was already defined in the derivation we did in class.
- pages 71-76 absolutely core material - achieve 100% understanding of this stuff
- pages 80-83 a very nice set of comments on the potential
- pages 88-91 boundary conditions! Are these new or already familiar?
- page 91-97 This section on energy is worth reading once and then reading again as he leaves some important comments to the end. Why does that surface term drop out?
- pages 97-107 conductions and capacitors - more core material - familiar already? Any surprises?

#### Material of note:

- The characters and their relations: Griffiths has introduced a number of “new” objects  $\mathbf{E}, V, \rho, \dots$  Describe in physical terms what these things are (be conservative in these definitions) then describe the mathematical relations.
- Gaussian surfaces: Use the argument in the three situations including problems 2.11 and 2.13.
- Define electrostatic energy. Include careful discussions of the “inconsistency” between equations 2.45 and 2.42 and on where the energy is stored. Why does that surface term drop out? Consider 2.34.
- Discuss conductors - properties and induced charges. Read the articles in the “By the way” footnote on page 99.

- Discuss boundary conditions of the  $\mathbf{E}$  and  $V$  and the section 2.5.3