Electrodynamics (PHYS 480) Second Guide

Fall 2014 v1.0

As I mentioned class, Chapter 2 is the first of what will be five chapters on the kinematics of electromagnetism. This study of electro- and magneto-statics will comprise about half the material of the seminar. And we start with the most familiar electric field. "The Problem" that we are faced with (or, rather, electrodynamics is faced with) is stated in the first paragraph: *Given a set of charges, what is their motion?* As Griffiths argues the key step in solving this problem is finding the electromagnetic fields. Many of the techniques introduced in this and later chapters help us with this problem.

The material of Chapter 3, *Laplace's equation, image charges, separation of variables, and the multipole expansion* is core stuff for 480. (BTW Ooops, I think I confused the names of the Poisson and Laplace equations last Thursday. Laplace is the $= 0$ one. Sorry!) This may be the first or second course in which you have the delight of using all sorts of orthogonal functions (e.g. Fourier, Legendre, Bessel, ...). (These are also discussed in Phys 320.) As the week progresses, we'll discuss where we are and whether we should speed up, slow down, or keep the pace as is. In any case I recommend pushing along on the reading through Chapter 3 over the weekend.

On Monday we'll finish up a bit of chapter two material on capacitors, and run through the image charge method, before returning to the solution of the Laplace equation.

There is a good chance that I will leave early on Friday, after a lunchtime meeting. So I will move my office hours to the morning and suggest starting work on the problem early!

I have ordered the problems in order of our discussion so that the ones you can already answer are at the beginning. In fact as of now (Friday) we have discussed the material for all of these except the last one.

Problems: Due Friday September 19.

(1) Combine the time derivative of Gauss' law and the divergence of Amperé's law with Maxwell's correction (the fourth line of Eq. (1) on the first guide) to find an equation which relates charge density ρ to the current density J. What is this equation called?
(2) 2.6 The $\vec{E}$ field of a plane of charge (Feel free to use your result of 2.5 from last week.)
(3) 2.17 Gauss' law with planar symmetry.
(4) 2.31 Finding the energy required to assemble charges
(5) 2.39 Playing with charges and conductors
(6) 2.41 Electrostatic pressure between plates.
(7) 3.3 Simple Laplace's equation in spherical coordinates.
(8) 3.15 This is a variation on the rectangular configuration of example 3.3. I recommend that you start from scratch .
(9) 3.24 Try out solving Laplace's equation in cylindrical coordinates.
(10) 3.6 Image charges!

Notes on text:

- page 115-7 The two points, repeated in different dimensions are *the* key points to Laplace's equation.
- page 119 On (the physical) Uniqueness of Solutions: To find the electric field within a closed surface, we solve Laplace's equation in the interior subject to boundary conditions. One can specify the value of the potential itself (called "Dirichlet" boundary conditions). Or, one can specify the value of the normal derivative of the potential (called "Neumann" boundary conditions). Of course, we can mix and match on different parts of the surface as long as every part of the surface has one and only one type of boundary condition.

To argue that the solution is unique we first assume that, to the contrary, we have two solutions, say V_1 and V_2. Defining the difference as $\phi = V_1 - V_2$, we have another solution (since Laplace's equation is linear, superposition applies). Now the idea is to show that ϕ vanishes everywhere. As both solutions have to satisfy the same boundary conditions, ϕ vanishes on the boundary. Even so it is best to start out by expressing the boundary conditions (ϕ or its normal derivative vanishes on the boundary) in a somewhat odd but very useful form

$$\int_{\partial v} \phi\ \hat{\mathbf{n}} \cdot \nabla\phi\ da = 0$$

where ∂v is the bounding surface of the volume v. The condition can be rewritten as

$$\int_{\partial v} \phi \nabla\phi \cdot d\mathbf{a}$$

on which we can use the divergence theorem to arrive at

$$\int_{v} \nabla \cdot (\phi\nabla\phi) d^3 = 0.$$

Using the chain rule and the fact that ϕ satisfies Laplace's equation we find

$$\int_{v} |\nabla\phi|^2 d^3x = 0$$

Now the integrand is (as they say in the business) "manifestly" positive. Thus, the only way for this equation to be satisfied is for ϕ to be at most a constant. That is, the original potentials V_1 and V_2 are identical up to a constant (i.e. choosing a zero). **Hooray!** As soon as we have found a solution – any way we like – we have found *the* solution!! (This argument is more general than the one presented in the text pp. 118-120.)

- page 124 The method of images is a key one to have available in your collection of tools. There is an "image charge" way of thinking. Try to find it.
- page 130-150 Pure core material: It is also great stuff once you "get in the groove." Separation of variables is a technique which is used over and over in much of physics. Work through Griffiths examples. If you like, after reading through Example 3.3 work through 3.4 and/or 3.5 without looking at the book. If you get stuck refer back to the book. Griffiths makes all his references to Mary Boas, a good math methods text.
- page 158 The electric field of a dipole is a useful thing to have at hand. There are a couple of often used limits, on the polar axis and in the equatorial plane. They both scale as $1/r^3$.