Electrodynamics (PHYS 480) Third Guide

Fall 2007 v1.0

We have a bit to finish up in Chapter 2 and then we will spend two weeks on the material of Chapter 3, Laplace's equation, image charges, separation of variables, and the multipole expansion. This is perhaps the first course in which you have the opportunity to use the mathematics (most of) you encountered in Phys 320. It is important and just gets more so if you continue on to quantum mechanics. Two weeks gives us time to more fully digest these techniques. In the first week we concentrate on uniqueness of solutions, image charges, and separation of variables. In the second week, we continue with separation of variables and the multipole expansion.

I would suggest reading through the chapter at least to the end of the separation of variables (page 144) in the first week. Then reread this same section for the second week and finish the chapter by reading the section on multipole expansion. As far as the problems, work the problems through 3.12 in the first week and then finish any ones we don't get to and the ones on multipole expansion for second week.

Problems of note: Starred (*) problems are required for the first week.

(1) 2.18 What is this a model of?
(2) * 2.35 From last week, on conductors
(3) * 2.36 Induced charges in a conductor with spherical holes
(4) 2.48 Try the first few parts of this.
(5) * 3.1 An integration with a nice result.
(6) * 3.2 Earnshaw's theorem. Don't get caught up by Griffiths "one sentence" request. Justify the result as best you can.
(7) * 3.3 In cases of high symmetry, the solutions, even in three dimensions, become one dimensional. The result ought to be satisfying.
(8) * 3.6 FInding the force with image charges
(9) 3.7 Finishing Example 3.2
(10) * 3.8 More image charges in a spherical geometry
(11) 3.9 This is very similar to the first image charge example.
(12) * 3.10 A nice example of how multiple image charges work
(13) * 3.12 This one and the next are variations on the rectangular configurations of the previous three examples. I recommend that you start from scratch on these. You need to do enough of these sort of problems so that it feels like integrating x^n.
(14) * 3.15
(15) 3.17 Note $P_0(\cos\theta) = 1$
(16) 3.18 Hint: Write $\cos 3\theta$ as $\cos\theta$ and $\cos^3\theta$ then match this new expression for the potential with the general expansion.
(17) 3.22 A classic Legendre polynomial problem. Note: $P_l(-x) = (-1)^l P_l(x)$.
(18) 3.23 The works in cylindrical symmetry (a problem for the fourth week!)
(19) 3.24 Using the results of 3.23
(20) 3.27 Practice with Eq. (3.99)
(21) 3.33 A short check of the equation
(22) 3.34 A little electrostatics, kinematics, and integration.
(23) 3.41 A neat result which we will use later.
(24) 3.45 Showing the structure in the multipole expansion
(25) 3.49

Notes on text:

- page 112 These two points are *the* key points to Laplace's equation. They come up again in 2 and 3 dimensions.
- page 116 On (the physical) Uniqueness of Solutions: To find the electric field within a closed surface, we solve Laplace's equation in the interior subject to boundary conditions. One can specify the value of the potential itself (called "Dirichlet" boundary conditions). Or, one can specify the value of the normal derivative of the potential (called "Neumann" boundary conditions). Of course, we can mix and match on different parts of the surface as long as every part of the surface has one and only one type of boundary condition. To argue that the solution is unique we first assume that, to the contrary, we have two solutions, say V_1 and V_2. Defining the difference as $\phi = V_1 - V_2$, we have another solution (since Laplace's equation is linear superposition applies). Now the idea is to show that ϕ vanishes everywhere. As both solutions have to satisfy the same boundary conditions, ϕ vanishes on the boundary. Even so it is best to start out by expressing the boundary conditions (ϕ or its normal derivative vanishes on the boundary) in a somewhat odd but very useful form
$$\int_{\partial v} \phi \, \hat{\mathbf{n}} \cdot \nabla\phi \, da = 0$$
where ∂v is the bounding surface of the volume v. The condition can be rewritten as
$$\int_{\partial v} \phi \nabla\phi \cdot d\mathbf{a}$$
on which we can use the divergence theorem to arrive at
$$\int_v \nabla \cdot (\phi\nabla\phi) d\tau = 0.$$
Using the chain rule and the fact that ϕ satisfies Laplace's equation we find
$$\int_v |\nabla\phi|^2 d\tau = 0$$
Now the integrand is (as they say in the business) "manifestly" positive. Thus, the only way for this equation to be satisfied is for ϕ to be at most a constant. That is, the original potentials V_1 and V_2 are identical up to a constant (choosing a ground). Hooray! As soon as we have found a solution – any way we like – we have found *the* solution!! (This argument is more general than the one presented in the text pp. 118-120.)
- page 121 The method of images is a key one to have available in your collection of tools. There is an "image charge" way of thinking. Try to find it.
- page 127-144 Pure core material: It is also great stuff once you "get in the groove." Separation of variables is a technique which is used over and over in much of physics. Work through Griffiths examples. If you like, after reading through Example 3.3 work through 3.4 and/or 3.5 without looking at the book. If you get stuck refer back to the book. Griffiths makes all his references to Mary Boas, a good math methods text. You may have a different text which discusses the same material...
- page 153 The electric field of a dipole is a useful thing to have at hand. There are a couple of often used limits, on the polar axis and in the equatorial plane. They both scale as $1/r^3$.

Material of note:

First Week:

- Earnshaw's Theorem: Discuss problem 3.2, offer your favorite proof of the theorem (Maxwell's?). See Weinstock AJP 44 (1976) 526.
- Image charges: Why does this technique work at all? (We start with completely different problems!) Present Example 3.2 and problem 3.9.
- Separation of variables: Introduction (cartesian coordinates)
- Fun facts about Legendre polynominals as in Phys 320: This ought to come with a 3-4 page handout. Use information from a diff eq'ns course, Phys 320, Abromowitz and Stegun, Maple ...

Second Week:

- Problem 3.18 from scratch (i.e. from 3.54) - to show the full separation of variables method.
- Problem 3.23 - Separation of variables in cylindrical coordinates
- Multipole expansion - Where does the wonderful Eq. (3.95) come from?
- Dipole electric fields: General form, special cases, and problem 3.49