

Sophomore Seminar 218, Spring 2004

Space: Its Light, Its Shape.

Ideas for Projects

There will be one assigned project for this course worth 30% of your course grade. The following is a list of possible topics for the project, but we also encourage you to design your own topic. If you would prefer to do a project with another student discuss how you would complete the project together and arrange to meet with us.

- Take one of the classic computer games such as Hexapawn, Cube, or Star Trek and change the rules so that the game plays on non-trivial topologies. Create a model of the new game. What are the interesting aspects? Illustrate with games in progress. Can the player who starts always win? If you enjoy coding, re-write the software to work with the new rules.
- Take some of the complex mathematics and/or physics ideas being presented and make them accessible to middle school students or high school students. Create a lesson plan and necessary learning materials. If possible, present the material at a local school.
- As done in the first chapter of Weeks for two-dimensional chess, create a three-dimensional game that illustrates the nature of a twisted three-dimensional torus. Explain how the properties of this twisted toroidal game would translate to physical properties if our universe is a twisted torus.
- Build a simple telescope (we can obtain a kit for you). Learn to use it to make simple observations of celestial objects such as the moon and the moons of Jupiter. Sketch your observations. Explain what you would observe if space were a torus with radii smaller than the size of the solar system.
- Read the Liminet *et. al.* article on dodecahedral space topology in the October 10, 2003 issue of Nature (and the expanded archive version). Prepare a report and give a presentation.
- Evaluate the measurement techniques from a modern perspective in *The Measure of All Things*. Relate this to today's "measurement of all things" as embodied by the recent cosmic microwave background measurements.
- Read Conway's ZIP proof of the Classification Theorem (Appendix C). Create a presentation that makes this proof understandable to your classmates. Compare parts of it to the standard proof as found in standard topology texts.

- We've seen one model of hyperbolic space in this course. The Poincaré Disc and Upper half plane models are two other models. Explore the similarities, differences, idiosyncrasies of these models. Try tiling the different models with the same hyperbolic tiling.
- A graph consists of a set of vertices (points) and edges (curves or lines) between some pairs of points. Certain graphs can be drawn on certain surfaces so that no edges cross (except where they share a vertex). A complete graph on n vertices has an edge between every pair of vertices. What does the Euler Characteristic tell us about drawing complete graphs without crossing on surfaces?
- Learn about the Figure 8 Klein bottle. Create a model of the Figure 8 and the traditional Klein bottle. Prove that these are really the same.