Space: Its Light, Its Shape.

Chapter 12: The Gauss-Bonnet Formula and the Euler Number

Assignment: For Monday, February 21, 2005

- Read Chapter 12.
- Though everyone is responsible for reading all of the material and for working out all of the exercises, teams have been assigned specific material and exercises for which they are responsible in class presentations. You may want to come to class early to firm up and smooth out the exercises with your teammates.

Team 3: Define a cell division of a surface and provide a few examples.

Team 4: Present Exercise 12.1.

Team 1: Show the notation for the number of vertices, edges, and faces in a cell division. Show the results in a couple of examples.

Team 2: Present Exercise 12.2.

Team 3: Present Exercise 12.3.

Team 4: Present Exercise 12.4.

Team 1: Looking ahead, tell us the Gauss-Bonnet formula and the Euler formula for a surface.

Team 2: Provide the outline of how we will "discover" the Gauss-Bonnet formula for the sphere.

Team 3: Carefully show details of how to use cell divisions to find the Gauss-Bonnet formula for the sphere.

Team 4: Present Exercise 12.5.

Team 1: Show how to use what we have already done to discover the Gauss-Bonnet formula for the projective plane.

Team 2: Present Exercise 12.6.

Team 3: Present Exercise 12.7.

Team 4: Present Exercise 12.8 and 12.9.

Team 1: Present Exercise 12.11.

Team 2: Summarize the Gauss-Bonnet formulas for surfaces with different homogeneous geometries using the Euler number χ.

Team 3: Carefully show how to compute the Euler number of the connected sum of n tori.

Team 4: Present Exercise 12.13.

Team 1: Describe how we can easily distinguish all surfaces. Can we use a similar method to distinguish all three-manifolds?

Team 2: Present Exercise 12.14.

Team 3: Discuss the concept of curvature for a more general surface with homogeneous geometry. Why does a larger sphere have a smaller curvature?

Team 4: Present Exercise 12.15.

Team 1: Present Exercise 12.16.

Team 2: Redefine curvature of a sphere without using the word "radius" and extend this definition to general hyperbolic planes and

Team 3: Describe Gaussian Curvature and give the general Gauss-Bonnet Formula.

Team 4: Present Exercise 12.18.

Team 1: Present Exercise 12.19.

Team 2: Give and explain the formula relating the angle sum of a triangle to its area in a homogeneous geometry of constant curvature k.

Team 3: Present Exercise 12.20.

Team 4: What do we mean by "net total curvature" and how does it apply to non-homogeneous surfaces?

Team 1: Present Exercise 12.21.